Pre-readings and exercises for the summer in Economics

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Dear PPE students,

Welcome and congratulations on your offer!

Please find below the reading list and the mathematics exercises that we would like you to go through over the summer.

The economics part gives you some references that you can explore based on your interests, to help you for the tutorials and essays.

The mathematics part summarises the main mathematics concepts that you should have seen by the end of the A-levels. We will build on them in the economics and statistics courses. The exercises should help you identify what are your strengths and weaknesses. Please use the summer break to work through them and bridge any gap you may have in your knowledge. This will allow you to start the year on solid mathematical foundations and will free more time for you to focus on the new economics concepts that we will introduce in our weekly tutorials, rather than struggle on calculations.

If you want, you can write down the solutions and hand them in for feedback when we meet at the beginning of the year. We'll also discuss what you thought of these exercises and your potential questions.

Best wishes,

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I Suggested Readings in Economics

It is suggested that you (1) read at least one of the "Popular" books or three journal articles and (2) go over Unit 1 and Unit 2 of the CORE textbook. Please note that the popular books are not "required" reading and you **do not** have to purchase these books. Readings from the journals and the CORE textbook are freely available.

I.1 "Popular" books

- Why Nations Fail by D. Acemoglu and J. Robinson (Profile, 2013)
- The Undercover Economist by Tim Harford (Abacus, 2006)
- Poor Economics: A Radical Rethinking of the Way to Fight Global Poverty by A. Banerjee, and D. Duflo, (Penguin, 2011)
- Freakonomics by S. Levitt and S. J. Dubner (Penguin, 2005)
- End this Depression Now by Paul Krugman (Norton, 2012)
- The Price of Inequality by Joseph Stiglitz (Norton, 2012)
- Thinking, Fast and Slow by Daniel Kahneman (Penguin, 2012)
- A Random Walk Down Wall Street by Burton Malkiel (Norton, 2016)
- What Money Can't Buy by Michael Sandel (Penguin, 2013)
- Naked Statistics by Charles Wheelan (Norton, 2011) Chapters 1-5 & 7-10 (not economics, but will be helpful in your statistics course).
- Gambling on Development: Why Some Countries Win and Others Lose by Stefan Dercon (2022)

I.2 Journal articles

- Prendergast, Canice. 2017. "How Food Banks Use Markets to Feed the Poor." *Journal of Economic Perspectives* 31 (4): 145–62. Link Here
- Oates, Wallace E., and Robert M. Schwab. 2015. "The Window Tax: A Case Study in Excess Burden." *Journal of Economic Perspectives* 29 (1): 163–80. Link Here
- Dragusanu, Raluca, Daniele Giovannucci, and Nathan Nunn. 2014. "The Economics of Fair Trade." *Journal of Economic Perspectives* 28 (3): 217–36. Link Here
- Slonim, Robert, Carmen Wang, and Ellen Garbarino. 2014. "The Market for Blood." *Journal of Economic Perspectives* 28 (2): 177–96. Link Here
- Bernanke, Ben S. 2013. "A Century of US Central Banking: Goals, Frameworks, Accountability." *Journal of Economic Perspectives* 27 (4): 3–16. Link Here
- Sandel, Michael J. 2013. "Market Reasoning as Moral Reasoning: Why Economists Should Re-engage with Political Philosophy." *Journal of Economic Perspectives* 27 (4): 121–40. Link Here

- Mankiw, N. Gregory. 2013. "Defending the One Percent." *Journal of Economic Perspectives* 27 (3): 21–34. Auerbach, Alan J., William G. Gale, and Benjamin H. Harris. 2010. "Activist Fiscal Policy." *Journal of Economic Perspectives* 24 (4): 141–64. Link Here

- Auerbach, Alan J., William G. Gale, and Benjamin H. Harris. 2010. "Activist Fiscal Policy." *Journal of Economic Perspectives* 24 (4): 141–64. Link Here

- Moore, Tyler, Richard Clayton, and Ross Anderson. 2009. "The Economics of Online Crime." *Journal of Economic Perspectives* 23 (3): 3–20. Link Here

- Backhouse, Roger E., and Steven G. Medema. 2009. "Retrospectives: On the Definition of Economics." *Journal of Economic Perspectives* 23 (1): 221–33. Link Here

- Leape, Jonathan. 2006. "The London Congestion Charge." *Journal of Economic Perspectives* 20 (4): 157–76. Link Here

- Einav, Liran, and Leeat Yariv. 2006. "What's in a Surname? The Effects of Surname Initials on Academic Success." *Journal of Economic Perspectives* 20 (1): 175–87. Link Here

- Siegel, Jeremy J., and Richard H. Thaler. 1997. "Anomalies: The Equity Premium Puzzle." *Journal of Economic Perspectives* 11 (1): 191–200. Link Here

I.3 Principal Reading and Textbooks

Microeconomics and Macroeconomics

– CORE Economics, The Economy, Oxford University Press, and freely available online at https://www.core-econ.org:

Microeconomics

– Frank, R., Microeconomics and Behaviour, McGraw-Hill

– Varian, H. R., Intermediate Microeconomics: A Modern Approach, Norton

– Morgan, W., Katz, M. L. and Rosen, H. S., Microeconomics, McGraw-Hill

Macroeconomics

– Jones, C.I., Macroeconomics, Norton

Maths

– Ian Jacques, Mathematics for Economics and Business, Pearson (introductory)

– Malcolm Pemberton and Nicholas Rao, Mathematics for Economists: An Introductory Textbook, Manchester University Press (more advanced)

II Mathematics review

This mathematics review lists the topics that you have seen in high school that you should understand and master, as we will build on them in our economics tutorials. By making sure the mathematics foundations are solid before term starts, we will be able to dedicate more time to economics and the intuition, rather than spend time on calculations in our tutorials.

You have taken mathematics for the GCSEs or A-levels, therefore, you should be able to do basic calculations and algebra, find a derivative or plug in numbers in an equation, without spending too much time. Over the summer, try to get as much practice as you can on these things for them to become second nature. The exercises below serve as a guide on what to review but if you see that you struggle on some of them, feel free to find additional resources in A-levels textbooks to practice more.

You should learn to check your answers by yourselves. One very useful tool is Geogebra. You can plot curves, find the intersection of two lines, compute areas, etc. For most of the exercises below you can check your answers with this tool, and maybe you'll find it useful to use in your economics courses as well!

So that you see where we are going with all these maths, there is a short exercise at the end that link these concepts with economics.

If you want, you can write down the solutions and hand them in for feedback when we meet at the beginning of the year. We'll also discuss what you thought of these exercises and your potential questions.

II.1 Geometry

Concepts to review:

- What is a plane, what is the x-axis and the y-axis (traditionally)?
- How do you plot a point A given by the coordinates (2,1) in the x-y-plane?
- How do you relate the slope of a line and the gradient? How do you compute the slope of a line if I give you (i) the equation of the line (ii) two points through which the line passes?
- What's the gradient of a horizontal line in a plane? Of a vertical line?

II.2 Algebra

Calculus

Concepts to review:

- Simplification
- Development
- Factorisation

- The famous formulas:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$

- More generally, the binomial expansion

Exercise 1:

Simplify the expressions below (develop, factorise, whatever makes them nicer than what they look like now). If you cannot solve some of these questions in less than 1 minute, practice more by reviewing thoroughly the concepts above.

(i) $x^2 + 8x + 16$

(ii) $9x^2 - 30x + 25$

(iii) $36 - 49x^2$

(iv) $(14 - 7x)(14 + 7x)$

(v) $(2x + 3)^2$

(vi) $(x-3)(x+3) + (2x+3)^2 - 2x(2x+5) + 1$

Roots
Concepts to review:

- Solve a linear equation of the type $ax + b = 0$. How do you usually check your answer?
- Solve a quadratic equation of the type $ax^2 + bx + c = 0$, either by factorising, by spotting an obvious root or by computing the discriminant. When should we use one relative to the other?

Exercise 2:

Solve the equations below. Again, if you struggle, I would suggest to review your formulas and calculus. To check your answers, go to geogebra.org/classic, plot the equation $y = ax+b$ or $y = ax^2+bx+c$ in the xy plane (for instance, for the first one, you should type in $y = 7x+4$) and see when it is intersecting the x-axis (aka the horizontal line). The value of the x-coordinate of the point at the intersection is the solution.

(i) $7x + 4 = 0$

(ii) $-9x + 3 = 0$

(iii) $-\frac{1}{18}x - 2 = 0$

(iv) $4x^2 + 4x + 1 = 0$

(v) $x^2 + x - 2 = 0$

(vi) $x^2 - 169 = 0$

(vii) $2x^2 + 8x - 42 = 0$

(viii) $2x^2 + 3x + 7 = 5x^2 + 4x - 3$

System of equations

Review how to solve a system of linear equations with two unknowns. You should have seen two methods: by substituting one of the unknowns and by elimination.
Practice on the exercises below. Make sure you can solve with the two methods each time.
To check your answers, you can still use Geogebra. For instance, in the first example below, type in $2y - 4x = 2$, which is the line corresponding to the first equation. Then type $y = -3x + 6$ which corresponds to the second. You will see that the two lines intersect. Use the help to define the point A as the intersection of these two lines. We are looking for the value of the coordinates of point A. Solve this manually, and then check that the coordinates match what Geogebra tells you!

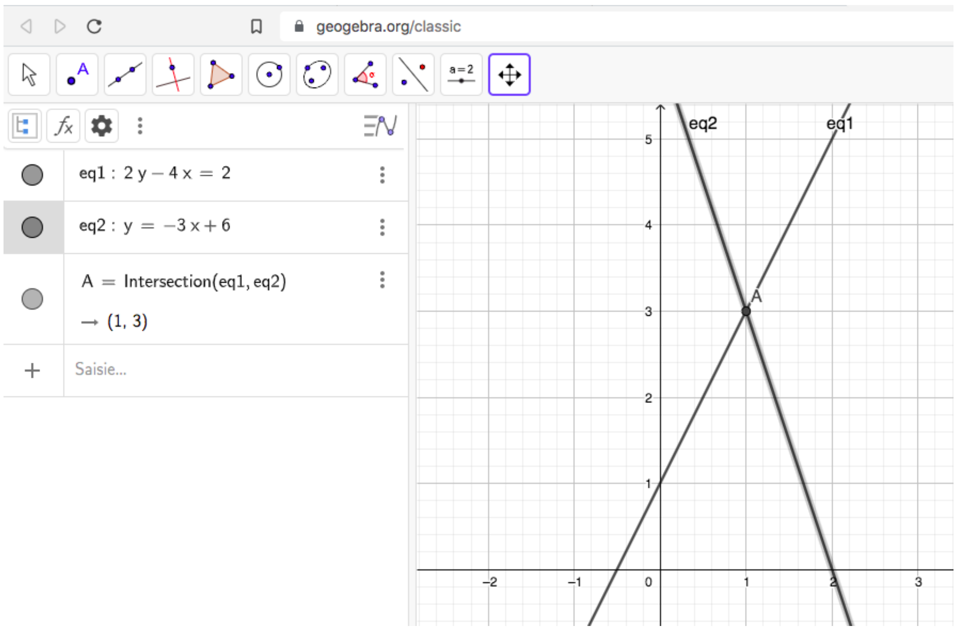

Figure 1: Example of how to solve exercise 1 on Geogebra

Exercise 3:

(i)
$$\begin{cases} 2y - 4x = 2 \\ y = -3x + 6 \end{cases}$$

(ii)
$$\begin{cases} 2x - y = 1 \\ 3x + 5y = 21 \end{cases}$$

(iii)
$$\begin{cases} 12x + 7y = 41 \\ 4x - 15y = 239 \end{cases}$$

(iv)
$$\begin{cases} \frac{1}{10}z + \frac{1}{20}k = 1 \\ \frac{2}{5}z - \frac{1}{10}k = 10 \end{cases}$$

II.3 Functions

Review of classical functions:

Whatever your mathematics background, you should know the following concepts:

- What is a function? Especially, understand that x in the formula $f(x) = ax + b$ is a mute variable, and that we can replace it by any other variable we want!
- Linear function: what is the definition? What is the meaning of the slope? In the graph with x labelling the horizontal axis and y labelling the vertical axis, how do you find the y-intercept and the x-intercept? How do you determine the equation for a linear function knowing that it goes through two points? How do you know whether it is increasing or decreasing? What is usually the interpretation?
- Quadratic function: Can you give me the definition? How do you plot them? How do you find the roots? How do you differentiate them? What is the domain and what is the range?
- Do you know the domain and the range of the following functions: linear, quadratic?
- Can you plot these functions in the x-y plane?
- Can you differentiate these functions?
- Can you integrate these functions over a specific interval?
- Can you find the interval over which these functions are positive, negative, or above whatever constant I give you?
- For linear functions of the type $f(x) = ax+b$, can you tell me what happens if (i) a increases or decreases while keeping b constant (ii) b increases or decreases while keeping a constant?
- For quadratic functions of the type $f(x) = ax^2 + bx + c$ can you tell me what happens if c increases?

Exercise 4:

There is no secret here. If you want to get better at this, you need to learn to draw these functions without using a calculator. I would recommend you try these exercises by yourselves, without a calculator, and then you check your results using Geogebra. Plot the following graphs in the x-y plane (so x on the horizontal axis and y on the vertical axis). This should be revisions and you should be able to do this quickly (i.e. no more than 5 minutes for the first four, no more than 10 for the last ones).

(i) $y = 3x + 4$

(ii) $y = -4x + 7$

(iii) $3y + 5x = 4$

(iv) Plot the function $f(q) = 15q + 3$

(v) $y = -2x^2$

(vi) $y = (x - 3)^2$

(vii) $y = -4x^2 - 4x + 24$

(viii) Plot the function $g(q) = (7 - q)(3 + q)$

Classical functions: The main classical functions are inverse, square root, exponential, logarithm. For each of these functions, you should review the following (i) what is their domain (ii) what is their range (iii) what is their derivative (iv) what is the anti-derivative (v) what is the graph of these functions

Exercise 5:

For each of the following functions, answer the questions (i) to (v) mentioned above (ideally without using a calculator!):

(i) $y = \frac{1}{x}$

(ii) $y = \sqrt{x}$

(iii) $y = e^x$

(iv) $y = ln(x)$

More complicated functions:
Review the following:

- Derivation: of a sum, difference, product, quotient.
- Integration: review the classical anti-derivatives.
- How do you relate the integral and the area between a curve and the horizontal axis?
- Review the fundamental theorem of calculus.
- Review the integration by substitution and the integration by part.

Exercise 6:

Differentiate the following functions. A good exercise would be to identify where it's defined and where it's not!

(i) $f(x) = -4x + 10$

(ii) $g(x) = xe^{2x}$

(iii) $h(x) = \frac{1}{x}ln(-3x)$

(iv) $i(x) = \frac{e^{-3x}}{7x^2+5x-3}$

(v) $j(x) = \sqrt{3x}\sqrt{5x^2+3}$

(vi) $k(x) = (2x+3)(-4x+7)e^x$

Exercise 7:

For each function below, think about where they are defined, integrate these functions and think about where the integral is defined. Then, compute each integral between the bounds given. For the last part, you can also check your answer using Geogebra, and I'll let you figure this out using the help.

(i) $f(x) = -4x + 10$ between 7 and 10

(ii) $g(x) = e^{2x}$ between 0 and 1

(iii) $h(x) = \frac{1}{3x}$ between 8 and 24

(iv) $i(x) = (7x^2 + 5x - 3)$ between 9 and 32

(v) $j(x) = \frac{1}{3x-4}$ between 3 and 6

(vi) $k(x) = 7x^4 - 4x + 2$ between 1 and 2

Note: The following section is slightly harder, so do not worry if you struggle.
As for simple functions, the best way to see whether you master these topics is to plot functions. First give it a try without using a calculator, then check your answers with Geogebra, and try to understand where your mistakes came from. For very complicated functions, here are a few pointers on how to plot them really well:

- Find the domain where the function is defined (and, if you know, continuous)
- Find the domain where the function is differentiable
- Find the derivative and examine where it is equal to zero.
- Study the sign of the derivative $\rightarrow$ what is the link between derivative / gradient and the graph of the function?
- Find the x-intercept of the function (is this possible?)
- Find the y-intercept of the function (is this possible?)
- For those of you who have seen this, look at the limits in $+\infty$ and $-\infty$ or at the points where the function is not continuous. Do not worry if this is not familiar to you, we will see this in due course!
- If you need, get a few points to see where the function should go through
- Plot!

Exercise 8:

Sketch the curve of the functions below, following the guidelines given above:

(i) $f(x) = 3x^2(2x + 4)^2$

(ii) $g(x) = xe^{1+3x}$

(iii) $h(x) = ln(\frac{1}{3+7x})$

(iv) $i(x) = \frac{1}{1+3x}$

II.4 Sequences and series

Review the following:

- What is a sequence? What is a recurrence relationship?
- What is an arithmetic sequence? What is a geometric sequence?
- For arithmetic or geometric sequences, can you give the following:
  - (i) The recurrence relationship
  - (ii) The formula of the n^{th} term as a function of the first term
  - (iii) The sum of the first k terms

Exercise 9:

(i) We give the following progression: 100, 50, 25, 12.5, ... What type of sequence is this? Find its n^{th} term and the sum of the first 10 terms.

(ii) Same questions for the following progression: 5, 13, 21, 29, ...

(iii) We give the following progression: 4, 12, 36, ... Which term is the first to exceed 1 000 000?

II.5 Probability and statistics

Concepts to review from the A-levels: Probabilities:

- What is the sample space? What is an event? What is the event space?
- What are mutually exclusive events? What are independent events? What's the complement of an event?
- What is a Venn diagram?
- Traditional formulas: conditional probability formula, probability of the complement of A, probability of A union B.

Statistics:

- What is a normal distribution? What are the main parameters? What is its shape? How do you find probabilities using the normal distribution?
- How would you conduct a test for the mean of a Normal distribution with known, given or assumed variance?

Exercise 10:

An urn contains one green ball and one red ball. Three times we will pick one ball at random, write down its colour, and put it back in the urn. Let us introduce the following events:

- A is that the second ball we pick is red.
- B is that there are at least two green balls in our picks.
- G: "the ball picked is green" and R: "the ball picked is red"
- *Example of notation:* If we first pick a green ball, then a red ball and finally a red ball, we will write this down as "GRR"

1. Produce a Venn Diagram, indicate the events A and B and what combinations of draws are inside each event.
2. What is $\mathbb{P}(A)$? $\mathbb{P}(B)$?
3. What is $\mathbb{P}(A \cap B)$? $\mathbb{P}(A \cup B)$?
4. What are the events that are not in $A \cup B$?

II.6 Going beyond – Link with economics

Try to solve the exercise below. I've added clues at the next page if you get stuck but try to solve it without them first!

We are going to model the demand and the supply as follows. Let p denote the price of the good, and q the quantity of the good. We are assuming that the demand and the supply curves are both linear. This means that the equation for the demand curve will be of the form $p = a \times q + b$ and the equation for the supply curve will be of the form $p = c \times q + d$ where we need to determine the coefficients a, b, c and d.

1. Suppose that consumers demand 10 biscuits when the price is 8 and 35 biscuits when the price is 3. What is the equation for the demand curve?

2. Suppose that producers will provide 5 biscuits if the price is 3 and 10 biscuits if the price is 5. What is the equation for the supply curve?

3. Plot the two lines in a graph, where q will be the horizontal axis, and p the vertical axis.

4. Find the equilibrium, that is the value of the price and the quantity at the intersection of the demand and the supply curves. Check your answers below, if you have the same graph, well done!

5. In economics, we are often referring to the consumer surplus and the producer surplus. We will give a more economics meaning in classes in Michaelmas Term Week 2, but for now let's stick to the following definitions:

   - Consumer surplus: the area below the demand curve and above the equilibrium price. Specifically, it's the green area in the graph at the next page.
   - Producer surplus: the area above the supply curve and below the equilibrium price. Specifically, this is the red area in the graph at the next page.

   Calculate the value of the consumer surplus and the producer in two ways:

   (i) Using the formula for the area of a triangle.

   (ii) Using integration!

As you have seen, this exercise uses many of the concepts we have seen above: calculations, solving a system of two equations, two unknowns, plotting a graph, geometry, integration etc. Hopefully this convinces you that it is necessary to have strong foundations in mathematics to think about the intuition.

Clues for exercise II.6:

1. That is you want to find the coefficients a and b such that $p = aq + b$. Plug in the numbers in the questions, this should be a system of two equations, two unknowns.
2. Same as above but this time the numbers are different!
3. The graph should look like this:

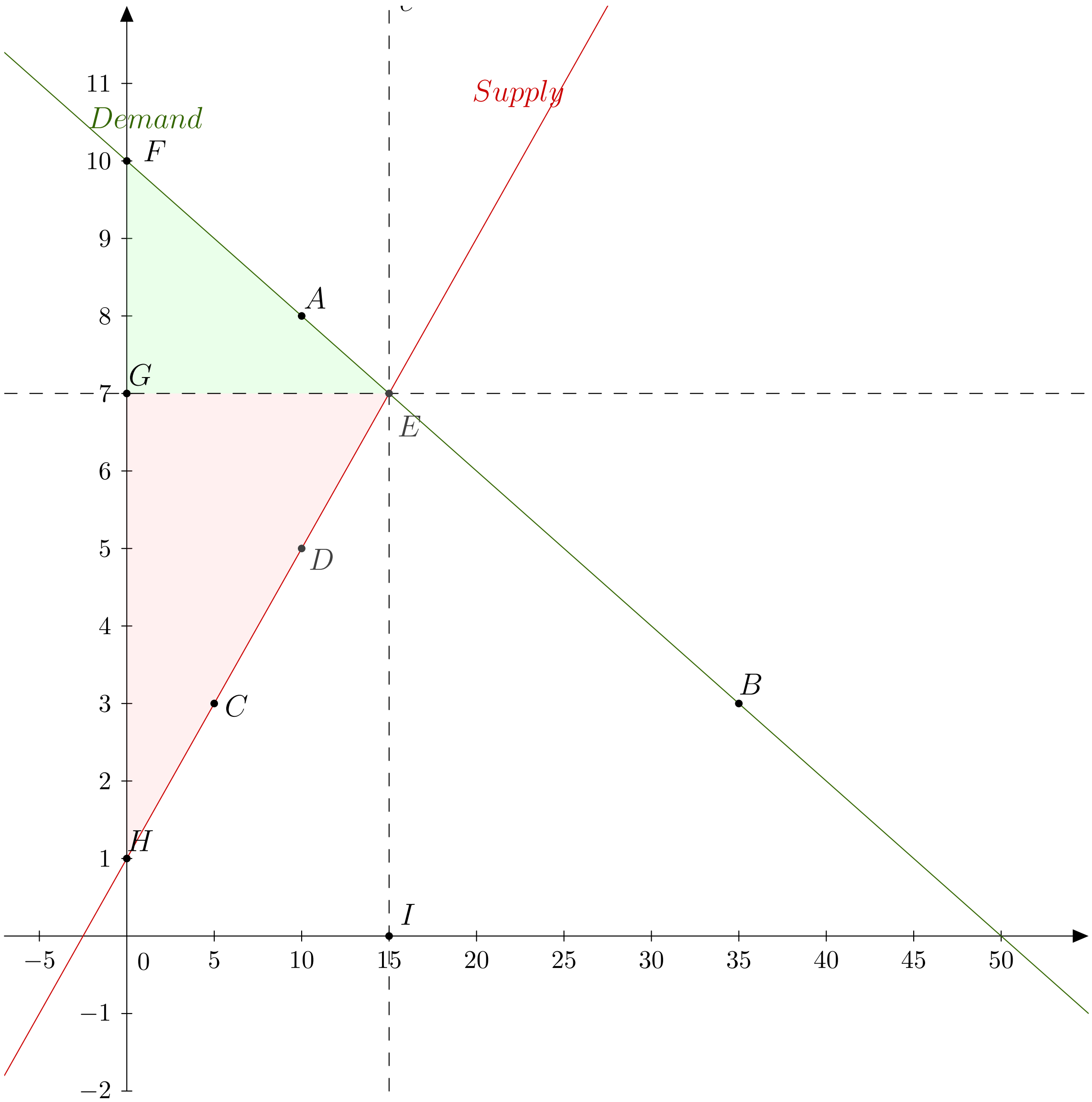

Figure 2: Graph of the demand and supply curve. The horizontal axis shows quantity, the vertical axis shows the price

4. Solution for demand and supply:
   - Demand: $p = -0.2q + 10$
   - Supply: $p = 0.4q + 1$.

   By definition, the intersection sets them equal. Find the q and then solve for p.

5. Hint for the consumer surplus (you should be able to do the same for producer surplus by yourselves!):

   (i) We're looking for the area of triangle EFG. What is the formula for the area of a triangle?

   $$Area = \frac{Base \times Height}{2}$$

   (ii) With integrals: decompose the area of EFG as the area below the demand curve (so FEIO) minus the area of the rectangle EGOI. How can you use the formula for the integral to compute the area FEIO?