

HEARINGS AND MISHEARINGS: DECRYPTING THE SPOKEN WORD

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We propose a model of the speech perception of individual words in the presence of mishearings. This phenomenological approach is based on concepts used in linguistics, and provides a formalism that is universal across languages. We put forward an efficient two-parameter form for the word length distribution, and introduce a simple representation of mishearings, which we use in our subsequent modelling of word recognition. In a context-free scenario, word recognition often occurs via anticipation when, part-way into a word, we can correctly guess its full form. We give a quantitative estimate of this anticipation threshold when no mishearings occur, in terms of model parameters. As might be expected, the whole anticipation effect disappears when there are sufficiently many mishearings. Our global approach to the problem of speech perception is in the spirit of an optimisation problem. We show for instance that speech perception is easy when the word length is less than a threshold, to be identified with a static transition, and hard otherwise. We extend this to the dynamics of word recognition, proposing an intuitive approach highlighting the distinction between individual, isolated mishearings and clusters of contiguous mishearings. At least in some parameter range, a dynamical transition is manifest well before the static transition is reached, as is the case for many other examples of complex systems.

Keywords: Speech recognition; Mishearings; Combinatorial Optimisation

1. Introduction

Any language is at once the vehicle of poetry and literature, and a precise algorithm for communication between its speakers, whose efficacy depends, among other things, on the size of its lexicon and the complexity of its grammar. Any aspect of language, be it its intrinsic structure or its decryption, is then as much a matter of science as of art: in this sense, the study of languages is one of the first instances of truly interdisciplinary academic activity. The analysis of languages accordingly includes approaches that range from being very descriptive and instance-specific, to

ones that are global and quantitative. An early instance of the latter concerns studies of the word length distribution in various texts and languages, whose historical roots date back at least to the middle of the 19th century (see Chapter 2 of [13] for a comprehensive review). Nowadays extensive databases such as the Leipzig Corpora Collection [10] are available, allowing a comparison between models and data.

The aim of this work is to model the alteration of spoken words by mishearings and the decrypting of speech in the presence of mishearings. Our approach is that of statistical physics. Earlier applications of concepts and tools from statistical physics to various aspects of linguistics are reviewed in [33, 21, 6, 2]. We endeavour to make a minimal model, keeping only the essential features of the problem. Among the most important linguistic concepts that we draw upon is the notion of ‘underspecification’ [25] in speech, which increases the efficiency of the decryption processes; this is done by incorporating an intermediate stage where mishearings are allowed for in the construction of a possible word.

The plan of this paper is as follows. Section 2 contains some preliminary material to be used in subsequent developments. We present our modeling of mishearings and the ensuing statistics of word variants (Section 2.1), as well as a very efficient two-parameter representation of the word length distribution across languages (Section 2.2). Our main results on speech decryption are presented in the two following sections. Section 3 is devoted to a phenomenological analysis of the statics of word recognition, including the highlighting of a static easy-to-hard transition (Section 3.1) and the analysis of the anticipation effect in the absence and in the presence of mishearings (Sections 3.2 and 3.3). A more intuitive investigation of dynamical aspects of word recognition is presented in Section 4. Section 5 contains a brief discussion of our findings. An appendix is devoted to the statistical mechanics of chains with intra-cluster interactions.

2. Spoken words and their variants

In the branches of linguistics devoted to spoken language, i.e., phonetics and phonology (see [13, 16, 22, 14] for overviews), the smallest distinctive unit of speech is called a phoneme. These phonemes, divided into vowels and consonants, are specific to a given language and range between a minimum of 11 and a maximum of 160 [16] across world languages. Typically, languages have 20 to 40 phonemes; English, for example, has 44.

Lahiri and co-workers [24] sought to make this deconstruction more universal, by relating perceived sounds to anatomical *features*, so that language-independent frameworks for analysis could be set up. The universality of this scheme provided the inspiration for our model below, which is also set up in terms of a language-independent formalism. That said, this first attempt is far from incorporating the subtle details of features. We refer instead to the elementary units of speech simply as ‘sounds’ which are meant to be universal across languages, along the lines suggested by Lahiri et al [24]. Another core assumption in our model is that the

linguistic lexicon is *structureless*, in the sense that the different sounds that form a word are not correlated among themselves. This provides a useful simplified framework for our investigations.

2.1. Mishearings and word variants

This section is devoted to our modeling of mishearings. For that purpose it is sufficient to deal with the simplest of all possible lexicons, where a word of length n ,

$$w = \sigma_1 \dots \sigma_n, \quad (1)$$

is nothing but an arbitrary sequence of n sounds, where each sound σ_i ($i = 1, \dots, n$) is chosen among the ν sounds of the language under consideration. Within this framework, the number W_n of ‘unique’ (i.e., distinct) words of length n ,

$$W_n = \nu^n, \quad (2)$$

grows exponentially with the word length. This oversimplified situation giving rise to an exponential proliferation of words will be replaced, from Section 2.2 onward, by the more realistic setting of a finite lexicon and a non-trivial, language-specific word length distribution.

We model mishearings as independent, random and local alterations of sounds. In the presence of mishearings, the spoken word w is thus heard as a ‘variant’

$$\tilde{w} = \tau_1 \dots \tau_n, \quad (3)$$

where each sound σ_i in the word w can either be misheard (with probability q) or correctly heard (with probability $1 - q$). The mishearing probability q is one of the key parameters of this work. We further assume for simplicity that each sound can only be misheard in one particular way. For instance, the sound ‘n’, if misheard, will always be perceived as ‘m’. These rules may be summarised as

$$\tau_i = \begin{cases} \phi(\sigma_i) & \text{with prob. } q, \\ \sigma_i & \text{with prob. } 1 - q. \end{cases} \quad (4)$$

The alteration function ϕ encodes the allowed mishearing of each sound; for instance, $\phi(\text{‘n’}) = \text{‘m’}$.

For a word w of length n , i.e., consisting of n sounds, if exactly k sounds at specified positions are misheard, the rules (4) generate

$$\omega = 2^k \quad (5)$$

distinct word variants \tilde{w} . For instance, for the word $w = \sigma_1 \sigma_2 \sigma_3$ of length $n = 3$, if the last two sounds are misheard, we have $k = 2$ and $\omega = 4$. The four variants read

$$\tilde{w} = \sigma_1 \sigma_2 \sigma_3, \quad \tilde{w} = \sigma_1 \phi(\sigma_2) \sigma_3, \quad \tilde{w} = \sigma_1 \sigma_2 \phi(\sigma_3), \quad \tilde{w} = \sigma_1 \phi(\sigma_2) \phi(\sigma_3). \quad (6)$$

The distribution of the number ω of variants of a given word w of length n ensues from the observation that the number k of mishearings is distributed according to

the binomial law:

$$\text{Prob}\{k\} = \binom{n}{k} q^k (1-q)^{n-k} \quad (k = 0, \dots, n). \quad (7)$$

One key quantity in subsequent developments is the average number of variants of a word of length n :

$$\Omega_n = \langle \omega \rangle = \sum_k 2^k \text{Prob}\{k\} = e^{\kappa n}, \quad (8)$$

with

$$\kappa = \ln(1+q). \quad (9)$$

The average number of variants per word thus also grows exponentially with the word length. The relationship between the numbers of words (see (2)) and their variants (see (8)) can be usefully expressed in terms of a scaling exponent δ , such that

$$\Omega_n \sim W_n^\delta, \quad (10)$$

i.e.,

$$\delta = \frac{\kappa}{\ln \nu}. \quad (11)$$

For typical parameter values such as $q = 0.2$ and $\nu = 20$, we obtain

$$\delta \approx 0.06. \quad (12)$$

The smallness of this value suggests that the number of variants is not overwhelmingly large, so that even with mishearings words are typically rather easy to recognise.

As an alternative to (8), one could instead consider the most probable number of variants of a word of length n , defined according to the usage in the statistical physics of disordered systems [5, 30]:

$$\bar{\Omega}_n = e^{\langle \ln \omega \rangle} = e^{\bar{\kappa} n}, \quad (13)$$

with

$$\bar{\kappa} = q \ln 2. \quad (14)$$

More generally, all the moments of the number ω of variants per word grow exponentially with the word length. We have indeed

$$\langle \omega^s \rangle = \sum_k 2^{sk} \text{Prob}\{k\} = e^{\lambda(s)n}, \quad (15)$$

where the exponent

$$\lambda(s) = \ln(1 + (2^s - 1)q) \quad (16)$$

depends non-linearly on the order s of the moment under consideration. Here s is not necessarily an integer. We have consistently $\kappa = \lambda(1)$ and $\bar{\kappa} = \lambda'(0)$, where the

prime denotes a derivative. A scaling law such as (15), with a non-linear dependence of the exponent $\lambda(s)$, is reminiscent of multifractal analysis [32, 37].

In the present situation, $\lambda(s) \approx (2^s - 1)q$ vanishes linearly with the mishearing probability as $q \rightarrow 0$, irrespective of the order s . This justifies the use of the average number Ω_n of word variants in the subsequent analysis of various quantities. Considering another moment of that number, including the most probable one ($s \rightarrow 0$), would indeed essentially amount to rescaling the mishearing probability q by a constant factor.

2.2. Word length distributions

In this section we introduce an efficient parametrisation of the word length distribution across natural languages, which will be used in what follows to refine the calculations presented in Section 2.1 of word variants generated by mishearings.

The study of the distribution of word lengths is an old subject (see Chapter 2 of [13] for a comprehensive review). An overwhelming majority of the available data concerns the length distribution of unique written words, where each word of the lexicon is counted once, and its length is defined as the number of its letters. A broad variety of parametrisations of the latter distribution has been proposed, ranging from the simplest (geometric, Poissonian, log-normal) to arbitrarily complex multi-parameter distributions. It appears that word length distributions in most current languages are characterised by two main features: a very rapid initial increase and a more or less steep decay with size. In this work we put forward the use of the Gamma distribution

$$p_n = C n^\alpha e^{-\beta n} \quad (17)$$

as an efficient way to describe these features. This distribution has a minimal number of two parameters, α and β , describing respectively the rise and the fall-off of the distribution. In spite of this simplicity, the Gamma distribution (17) has hardly ever been used to parametrise word length distributions in their generality (see [36, 11, 8] for its use in specific instances). In Figure 1, we demonstrate that it provides a very good parametrisation of the full word length distribution for four European languages (upper panel) and four Asian languages (lower panel), which have between them a broad range of average word lengths. The dashed curves show fits to (17), to be compared with empirical data (full curves) for the length distributions of unique written words provided by the Leipzig Corpora Collection [10]. The corresponding fitted parameter values α and β are given in Table 1. Fits of similar quality have been obtained across many other languages which we have considered. We therefore argue that the Gamma distribution (17) has a universal validity across most world languages.

In most languages, the word length distribution is observed to be asymmetric, with a rise steeper than its fall. As a consequence, the average word length \bar{n} is (slightly) larger than the most probable word length n_* . Furthermore, \bar{n} and n_*

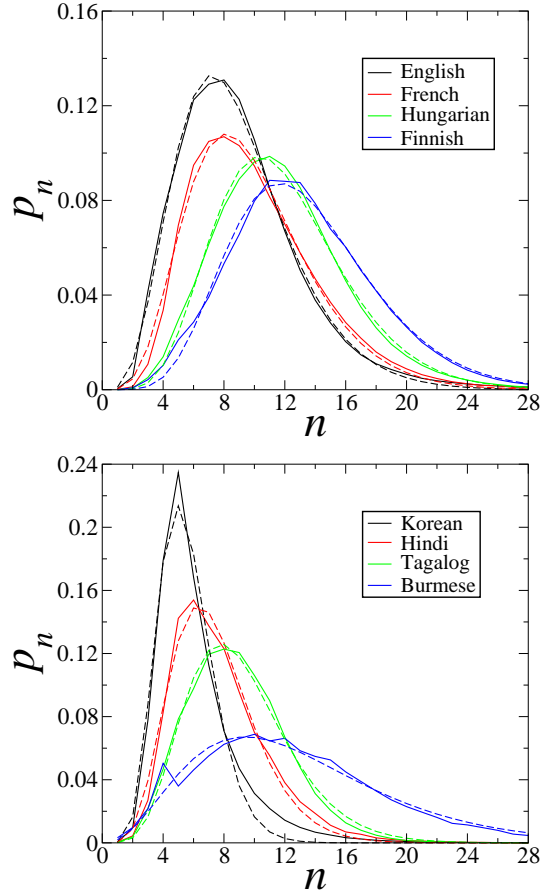


Fig. 1. Distribution of the length of written words for four European languages (upper panel) and four Asian languages (lower panel). Full curves: data from the Leipzig Corpora Collection [10]. Dashed curves: two-parameter fits to (17).

are large enough that they can be reasonably evaluated by treating n in (17) as a continuous variable. This yields $C = \beta^{\alpha+1}/\Gamma(\alpha+1)$ and

$$\bar{n} \approx \frac{\alpha+1}{\beta}, \quad n_* \approx \frac{\alpha}{\beta}, \quad (18)$$

where it is understood that the second estimate is rounded to the nearest integer. The resulting numerical values of \bar{n} and n_* are also given in Table 1 for all languages shown in Figure 1. For the European languages we have chosen, the average word length increases consistently from English through to Finnish, as shorter words are more and more suppressed; this is reflected by an increase of α , while β varies only mildly. On the other hand, for the Asian languages we have chosen, both α and β show appreciable variations.

Language	α	β	\bar{n}	n_*
English	4.4	0.60	8.3	7
French	4.9	0.60	10.1	8
Hungarian	6.7	0.64	11.9	10
Finnish	6.8	0.58	13.4	12
Korean	8.2	1.65	6.2	5
Hindi	6.0	0.94	7.7	6
Tagalog	6.0	0.73	8.8	8
Burmese	2.6	0.28	12.8	9

Table 1. Numerical values of the fitted parameters α and β entering the Gamma form (17) of the word length distribution, and of the resulting average and most probable word lengths \bar{n} and n_* (see (18)), for all languages shown in Figure 1.

For our purposes, the relevant concept of a word length is the number of its sounds, since we are here concerned with hearings and mishearings. Fortunately, it turns out that there is a high degree of correlation between the length distributions of written words (numbers of letters) and spoken words (numbers of ‘sounds’, which here represent phonemes), for most languages. As a matter of fact, sounds and letters enjoy a near one-to-one correspondence in ancient languages like Latin or Greek, so that their length distributions for written and spoken words are virtually identical. We would therefore expect that this high degree of correspondence between phonology and orthography would persist in modern languages; and have, in order to confirm this expectation, performed a preliminary analysis of this question in the case of three languages (German, Dutch and English), based on the CELEX database [3]. Our main conclusion is that there is a fixed ratio $r \approx 0.85$ between the numbers of sounds and letters in typical words [26]: we obtain nearly identical values of r for unique words (when every word of the lexicon is counted once) and token words (when words are counted according to their frequencies of occurrence). Furthermore, this value of the ratio r appears to have a high degree of universality and in particular, does not show any appreciable difference among the three languages we have tested.

We will, in what follows, therefore use the length distribution of written words (measured by numbers of letters and parametrised by the Gamma distribution (17)) as a proxy for that of spoken words (measured by numbers of sounds). Our framework of the modelling of mishearings and the ensuing statistics of word variants (derived in Section 2.1) will also be extensively used in the next sections, where we will continue with our assumption of a structureless lexicon. The only language-specific ingredients entering our work will be the parameters α and β in the parametrisation (17) of the word length distribution, which are listed in Table 1 for all languages shown in Figure 1.

3. The statics of word recognition

3.1. *Decrypting words – the easy-to-hard transition*

A key feature of speech decryption is that the number of variants induced by mishearings should not be too large. In this section we first define a threshold δ_n , which determines the difficulty of decryption in terms of a simple ratio involving the numbers of words and their variants. Another meaningful quantity which we introduce in this context is $\delta_{n_{\text{st}}}$, which describes the onset of word lengths which are so large that decryption becomes very difficult; this occurs both because the words themselves are exceptionally long and therefore rare, and because they generate huge numbers of variants. Finally, we introduce a mechanism by which most words are actually recognised: this is the phenomenon of *anticipation* where, part way through a word, a listener can guess what it actually is. In the concluding parts of this section, we quantify the phenomenon of anticipation: first, in the absence of mishearings, and then, in their presence. Our results demonstrate that the phenomenon of word anticipation is much more efficient in the former case compared to the latter, which is intuitively reasonable.

For a language with an overall lexicon size Λ and an arbitrary word length distribution p_n , we expect that it would be easy to recognise a word w of length n if the average number Ω_n of its variants is much smaller than the total number

$$W_n = \Lambda p_n \quad (19)$$

of words of length n . This criterion is conveniently measured by the effective exponent

$$\delta_n = \frac{\ln \Omega_n}{\ln W_n} = \frac{\kappa n}{\ln \Lambda p_n}. \quad (20)$$

This quantity has a rich non-linear dependence on the word length n . Its numerator grows linearly with n , as a consequence of (8), with κ defined in (9). Its denominator involves the full word length distribution, parametrised as (17), with language-specific parameters α and β . The introduction of the n -dependent scaling exponent δ_n is one of the novel features of our work, allowing for the application of concepts stemming from finite-size scaling. This demarcation of complexity by size is strikingly reminiscent of problems encountered in combinatorial optimisation [12, 34, 31, 18].

For the typical parameter values $q = 0.2$ and $\Lambda = 10^5$, the effective exponent δ_* , corresponding to the most probable word length n_* of all languages shown in Figure 1, is tabulated in Table 2. We note that, while δ_* is small, it is significantly larger than the corresponding value (12) for a free lexicon with no constraint on the word length distribution; this suggests that constraints make word identification more difficult, as might be expected.

For very long words, two effects appear. First, the length distribution p_n falls off very fast, so that the corresponding words are genuinely rare; second, the sheer length of the word generates hugely many variants in the presence of mishearings.

Language	δ_*	n_{st}	m_{ant}
English	0.14	25	4
French	0.16	26	5
Hungarian	0.20	27	7
Finnish	0.24	29	8
Korean	0.09	15	3
Hindi	0.11	20	4
Tagalog	0.15	23	5
Burmese	0.19	32	6

Table 2. Values of characteristic quantities pertaining to our static approach to speech recognition (see Sections 3.1 and 3.2), for all languages shown in Figure 1: δ_* is the effective exponent δ_n of (20) measured for the most probable word length n_* , whereas n_{st} is the static crossover length such that $\delta_{n_{\text{st}}} = 1$ (see (21)), and m_{ant} is the anticipation length in the absence of mishearings (see (25)). Word length distributions are parametrised according to (17), with fitted parameters α and β given in Table 1. Other parameter values are $q = 0.2$, $\Lambda = 10^5$ and $\varepsilon = 1/20$.

Eq. (20) shows that these parallel effects cause the effective exponent δ_n to increase rapidly with the word length n . Word recognition becomes very hard when n reaches the static crossover length n_{st} , such that

$$\delta_{n_{\text{st}}} = 1. \quad (21)$$

This crossover length, where speech recognition becomes suddenly hard, is analogous to the occurrence of the static easy-to-hard phase transition met with in typical hard combinatorial problems [12, 34, 31, 18]. Since word lengths are always finite, any potential phase transition is of course rounded to a crossover; also, and equally clearly, its exact location depends on the number chosen on the right-hand side of the definition (21). However, and despite these quantitative caveats, the analogy of the crossover (21) with the easy-to-hard combinatorial phase transition is striking.

The static crossover length n_{st} defined in (21) is also given in Table 2 for all languages shown in Figure 1: note that this is typically three times larger than the average word length \bar{n} in each case (see Table 1). Words that are equal to or larger than this value in length are in the tails of the distribution; while they are not numerous – typically no more than a few hundreds – their rarity adds to the complexity of their decryption, as mentioned above.

3.2. Word anticipation with no mishearings

How do we decrypt individual words? A major role is played by the phenomenon of *anticipation*, where we expect to hear a particular word on the basis of the first few sounds that we hear. In this section, we will assume that both speakers and listeners are perfect, i.e., that no mishearings occur, while in the following one, we will see what happens when listeners mishear what has been said.

Assume the listener has heard a string

$$s = \sigma_1 \dots \sigma_m \quad (22)$$

of m sounds. The string s coincides with the first m sounds of some set of words w of the lexicon of length $n \geq m$; we therefore say that it is a prefix of each of those words.

The number S_n of word prefixes of length n can be estimated by merely assuming that each prefix s has a small probability ε to be an exact word, with ε being of the order of $1/\nu$. This yields

$$S_n = \frac{W_n}{\varepsilon} = \frac{\Lambda p_n}{\varepsilon}. \quad (23)$$

The average number f_m of words with the string s as a prefix is given by:

$$f_m = \frac{1}{S_m} \sum_{n \geq m} W_n = \frac{\varepsilon}{p_m} \sum_{n \geq m} p_n. \quad (24)$$

As the number $m = 1, 2, 3, \dots$ of heard sounds increases, the number of remaining possibilities decreases sharply. Usually, one is able to guess the word at a certain point before the word has ended: we call this the *anticipation length* m_{ant} , at which the number of remaining words becomes of the order of one:

$$f_{m_{\text{ant}}} = 1. \quad (25)$$

The anticipation length m_{ant} is also given in Table 2 for all languages shown in Figure 1. This length is always smaller than the most probable word length n_* , as might be expected for an anticipation effect. This concept, to which we give a quantitative underpinning here, is referred to as the uniqueness point in linguistics, e.g. in cohort models [24, 28]. The strength of the anticipation effect is measured by the ratio m_{ant}/n_* . For the choice of $\varepsilon = 1/20$, and for the languages we have considered, this ratio is nearly constant, suggesting that we can predict a typical word about two-thirds of the way through it.

3.3. *Word anticipation in the presence of mishearings*

Everyday experience tells us that mishearings make it difficult to recognise, never mind anticipate, spoken words. We would therefore expect that with enough mishearings, the anticipation of words would disappear entirely, which is indeed what our formalism predicts.

From now on, we consider the situation where each sound is misheard with probability q , according to the rules (4). Assume the listener has heard a string

$$t = \tau_1 \dots \tau_m \quad (26)$$

of m sounds. This string may be the outcome of the random insertion of any number $k = 0, \dots, m$ of mishearings into an existing word prefix s of length m . For a fixed

number k of mishearings, the mean number of words w compatible with the heard prefix t reads therefore

$$g_{m,k} = 2^k f_m. \quad (27)$$

This expression can be visualised as a ‘hammock plot’ (upper panel of Figure 2) of $g_{m,k}$ against the prefix length m , for model parameters corresponding to English (see Table 1) and $\varepsilon = 1/20$. The number k of mishearings is denoted by the symbol colours. The number of words varies over so large a range that a logarithmic scale is needed.

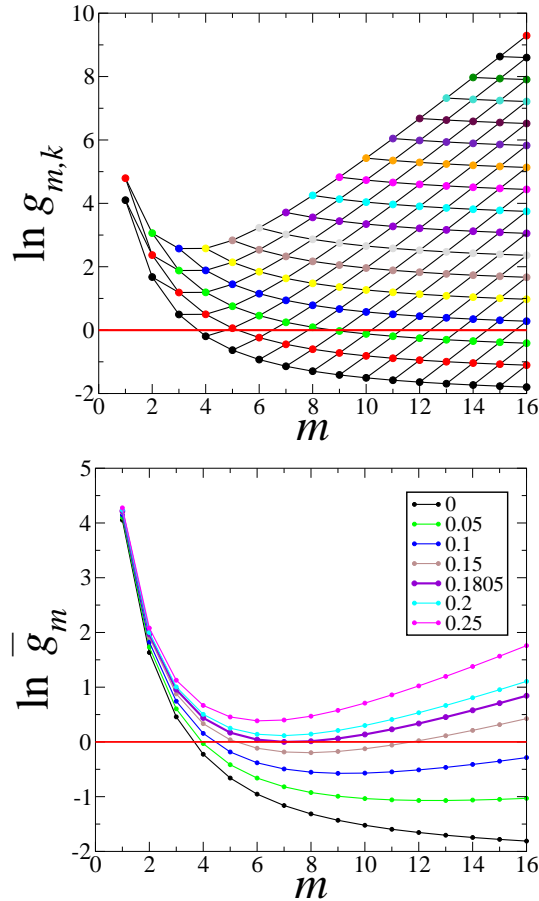


Fig. 2. Plots of $\ln g_{m,k}$ (upper panel) and of $\ln \bar{g}_m$ (lower panel) against prefix length m for model parameters corresponding to English (see Table 1) and $\varepsilon = 1/20$. In the upper panel, the number $k = 0, \dots, m$ of mishearings is denoted by the symbol colours (black, red, green, blue, etc. denote $k = 0, 1, 2, 3, \dots$). In the lower panel, colours denote values of the mishearing probability q (see legend). The thick purple curve corresponds to the threshold value $q_{\text{th}} = 0.1805$.

As before, the anticipation threshold is defined as the first point at which only one word remains possible. This amounts, in the present instance, to setting $g_{m,k} = 1$, which is shown in the upper panel of Figure 2 as a thick red horizontal line. Note that for the case of no mishearings, we have $k = 0$ and $g_{m,k} = f_m$, corresponding to the lowest branch of the plot (black symbols). The fact that this crosses the red line at $m = 4$ provides a consistency check with the fact that the onset of the anticipation effect for English in the absence of mishearings, m_{ant} , is indeed 4 (see Table 2).

The average number \bar{g}_m of remaining possible words after a sequence t of length m is heard can be evaluated by averaging the expression (27) over the known distribution of the number k of mishearings (see (7)):

$$\bar{g}_m = e^{\kappa m} f_m. \quad (28)$$

As the mishearing probability q increases, the exponential factor $e^{\kappa m}$ in the above expression increases rapidly, leading to a progressive weakening of the anticipation effect until it disappears above some threshold q_{th} . This threshold is defined as the point at which the minimum of \bar{g}_m over all prefix lengths m becomes unity. In the plot of $\ln \bar{g}_m$ against the prefix length m in the lower panel of Figure 2, this happens for the third curve from the top (thick purple curve), where the minimum grazes the red line corresponding to $\bar{g}_m = 1$. The corresponding threshold reads $q_{\text{th}} \approx 0.18$: for q less than this value (lower four curves of the plot), \bar{g}_m always crosses the red line, so that anticipation is possible even in the presence of mishearings. On the other hand, for q larger than q_{th} (upper two curves of the plot), \bar{g}_m stays above 1, so that there are too many variants for anticipation to be effective.

Table 3 gives the numerical values of the threshold mishearing probability q_{th} for all languages shown in Figure 1, as well as the corresponding values of the prefix length m_{th} where the minimum is reached at threshold and of the effective exponent δ_{th} . For all languages considered here m_{th} and n_* are nearly equal (compare Table 1 and Table 3), suggesting that the most probable word length n_* in a given language nearly optimises the possibility of decryption despite mishearings. The corresponding effective exponent δ_{th} is much less than unity, the value of δ at the static easy-to-hard transition $\delta_{n_{\text{st}}}$. This important distinction makes it clear that while $\delta_{n_{\text{st}}}$ epitomises the difficulty of recognising very long and complex words, δ_{th} corresponds to the befuddling effect of multiple mishearings, which can set in for words of even moderate sizes.

4. The dynamics of word recognition

In the above, we have only considered static aspects of word recognition. Here we turn our attention to the dynamics of this process: more specifically, what is the effect of mishearings on the time it takes for a listener to recognise a given word? Does it matter if these mishearings appear individually or if they are consecutive, and form a cluster of mishearings? Recalling that until now we have only addressed

Language	q_{th}	m_{th}	δ_{th}
English	0.18	7	0.12
French	0.15	9	0.14
Hungarian	0.12	12	0.14
Finnish	0.09	14	0.14
Korean	0.45	5	0.19
Hindi	0.26	6	0.15
Tagalog	0.18	8	0.14
Burmese	0.08	11	0.09

Table 3. Values of characteristic quantities pertaining to our approach to word anticipation in the presence of mishearings (see Section 3.3), for all languages shown in Figure 1: q_{th} is the threshold value of the mishearing probability at which the anticipation effect disappears, whereas m_{th} and δ_{th} are respectively the corresponding prefix length and effective exponent. Word length distributions are parametrised according to (17), with fitted parameters α and β given in Table 1. Other parameter values are $\Lambda = 10^5$ and $\varepsilon = 1/20$.

the total number of mishearings in a given word, with no positional information, the following intuitive approach at least gives us a hint of the difference between individual and collective impediments to word recognition.

We proceed in the spirit of an analogy with jammed granular media (see e.g. [29] and references therein), an example of an athermal and disordered system like our own. Prior to jamming in a shaken assembly of grains, a tracer grain can navigate its way past individual grains that only partially obstruct it; however, as the density increases, the obstacles represented by clusters of grains make it impossible for it to move beyond them, and this complete obstruction leads to jamming. In the present scenario, a low density of mishearings, whether isolated or contiguous, leads to a reasonable possibility of ‘escape’ for a listener from a situation of complete misunderstanding. In contrast, when there are many clusters of consecutive mishearings, the large number of variants essentially makes decryption impossible.

More specifically, we visualise the speech recognition algorithm as a random walker on a network with traps, where the latter correspond to mishearings. A low density of traps causes the slowing down of the random walker; as the density increases, the traps are more and more likely to occur contiguously, in clusters. In the latter case, dynamical arrest may result if the random walker has to traverse paths crossing arbitrarily large clusters of traps.

To reiterate – we have so far characterised speech recognition from a static viewpoint, focusing on the statistics of mishearings in a word w of length n . This was done by computing the average number Ω_n of variants per word of length n (see (8)), and the corresponding static exponents δ (see (11)) or δ_n (see (20)).

We now approach the problem from a dynamical point of view: how does a speech recognition algorithm react to a string of sounds that are being successively presented to it in real time? The analogies given above suggest that a cluster of

successive mishearings would clearly present a far greater obstacle to speech recognition than a few isolated ones. We therefore proceed by attributing a (dynamical) penalty, in the form of a statistical weight λ_k , to any set of k successive mishearings. This penalty should increase rapidly with the cluster size k , as larger clusters of mishearings will more significantly slow down the task of decryption. It should also be proportional to the complexity associated with the task of exploring such a cluster in an ordered way, and thus to the number of its possible permutations.

This intuitive line of reasoning leads us to associate cluster weights that grow factorially with a cluster of k consecutive mishearings:

$$\lambda_k = \mu^k k!. \quad (29)$$

The constant μ is a phenomenological parameter which cannot be estimated *a priori*.

The full dynamical complexity of deciphering a typical word of length n is thus represented by the partition function Z_n associated with a chain of length n in the presence of the cluster weights λ_k . From the formal definition of these quantities and their subsequent analysis in Appendix A, we obtain a logarithmic growth of the free energy density (see (A.19)):

$$K_n \approx \ln \frac{\mu q n}{e}. \quad (30)$$

The resulting superextensive growth of the total free energy $\ln Z_n = nK_n$ implies that the effective dynamical exponent,

$$\Delta_n = \frac{\ln Z_n}{\ln W_n} = \frac{nK_n}{\ln \Lambda p_n}, \quad (31)$$

grows more quickly with word length n than its static counterpart δ_n (see (20)). Proceeding to define the dynamical crossover length n_{dyn} by the condition

$$\Delta_{n_{\text{dyn}}} = 1, \quad (32)$$

we assert that for large enough word lengths and sufficiently high q , large clusters of mishearings will arrest the speech recognition algorithm at n_{dyn} , well before the static crossover length n_{st} is reached (see below).

The above ideas have a wealth of analogies in statistical and condensed matter physics. In addition to the dynamical arrest in granular media referred to above, similar dynamical transitions in diffusive motion have been widely observed in glasses (see e.g. [17, 27, 7, 20]), as well as via the occurrence of Anderson localisation, be this in disordered conductors (see e.g. [19, 1]) or in the multiple scattering of light (see e.g. [39, 35]). From a more theoretical point of view, a well-documented instance of slowing down from diffusive to sub-diffusive transport is provided by random walks on comb structures, in the regime where the mean value of the teeth depth diverges (see e.g. [15]).

In summary, our intuitive arguments above suggest that decryption in the presence of mishearings involves two successive thresholds as a function of word length n , viz. the dynamical length n_{dyn} (32) and the static one n_{st} (21). Both crossover

lengths depend on the lexicon size Λ , the mishearing probability q , and the language under consideration, through the parametrisation (17) of the word length distribution. A subtlety is that the dynamical crossover length n_{dyn} also depends on the unknown parameter μ . The upper limit of this dynamical length is naturally provided by the static length n_{st} , to which it becomes equal at a critical value μ_c . Whenever $\mu > \mu_c$, the dynamical length n_{dyn} exhibits a slow decrease as a function of μ ; at any particular value of the latter, the dynamical transition precedes the static transition, as it must. In the opposite regime ($\mu < \mu_c$), both static and dynamical transitions merge into a single one.

This is demonstrated in Figure 3 for two languages, Finnish and Burmese, which we have chosen because they have relatively high values of n_{st} (see Table 1). The dynamical length n_{dyn} varies over a broad range extending from the average word length \bar{n} to the static crossover length n_{st} , where the latter corresponds to word lengths deep in the tails of the distribution. For languages such as English, the picture is qualitatively similar, although the range over which n_{dyn} varies is smaller.

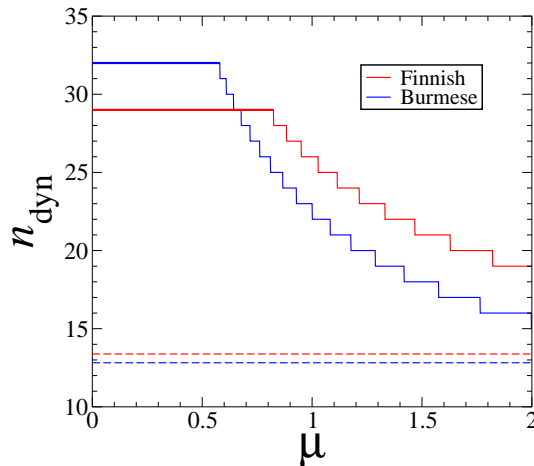


Fig. 3. Dynamical crossover length n_{dyn} against parameter μ , for Finnish and Burmese. Thick full horizontal lines: static crossover length n_{st} . Dashed horizontal lines: average word length \bar{n} . Word length distributions are parametrised according to (17), with fitted parameters α and β given in Table 1. Other parameter values are $q = 0.2$ and $\Lambda = 10^5$.

5. Discussion

We have in the above built a simple and intuitive model of speech perception. A unique feature of our model is the representation of mishearings via a single parameter, the mishearing probability q , providing a measure of the fraction of sounds that can be misheard in a given language. We go on to obtain intuitive

predictions in a series of scenarios relating to word recognition. In the absence of mishearings, we predict the existence of a threshold after which a word can be correctly guessed, viz. the anticipation length m_{ant} ; this provides a quantitative underpinning to a related concept in linguistics, the uniqueness point. We follow this with the consideration of anticipation in the presence of mishearings; as expected the phenomenon disappears above a threshold mishearing probability q_{th} .

A limitation of our current approach is that it proceeds by considering the fate of ‘typical’ words, thereby following a mean-field approach, where the role of fluctuations as well as correlations is neglected. Thus, we are so far unable to include positional information in our analysis – e.g. the relative difficulty of decryption depending on whether mishearings occur at the beginning or the end of a word [23], which is clearly important for real-world decryption. We aim to include the effect of such and other correlations in future work [26].

A leitmotif underpinning our work is the idea that speech perception is a problem of optimisation. We have identified a *static easy-to-hard transition* in terms of word length, after which spoken words are very hard to decipher. This occurs for words in the tails of the word length distribution, whose lengths are greater than the threshold static length n_{st} ; we have defined $\delta_{n_{\text{st}}}$ as a measure of the difficulty of decryption in this limit of long and typically rare words. In the presence of mishearings, however, decryption can be difficult for words of even moderate size; the quantity δ_{th} defined above embodies the difficulty of decryption in the presence of multiple mishearings.

The dynamics of speech recognition takes these ideas a step further, and examines the relative difficulties presented by isolated versus consecutive mishearings to word recognition: everyday experience tells us that clusters of consecutive mishearings are likely to provide far greater impediments to decryption than isolated ones. The dynamical complexity of such clusters increases with their size, a feature which is modelled by the association of appropriate statistical weights with them. The task of the speech recognition algorithm is to navigate these obstacles and correctly identify the word concerned, which becomes unfeasible beyond some dynamical length n_{dyn} , where clusters of contiguous mishearings can successfully arrest recognition. This phenomenon is associated with a *dynamical transition* which, as in the case of many other complex systems, precedes the static transition occurring at n_{st} .

We suggest that the dynamical transition is intimately associated with the phenomenon of ‘underspecification’ proposed by Lahiri et al [25]. Put simply, this involves the storage of all possible word variants corresponding to a set of mishearings in the listener’s memory, until an individual word is correctly recognised at a suitable point in a sequence of sounds. The presence of clusters of contiguous mishearings will cause this list to be hugely amplified, until accurate word recognition is impossible. Clearly, this can happen even for words close to the average word length \bar{n} , i.e., the dynamical transition n_{dyn} can set in for word lengths that vary between \bar{n} and the static crossover length n_{st} .

A possible experiment to test this would involve tests carried out on ensembles of

listeners with similar linguistic abilities in a given language, who would be subjected to ever-increasing clusters of contiguous mishearings in individual words; datasets would comprise words with lengths between \bar{n} and n_{st} . The objective would be to identify a threshold of incomprehension for most listeners which, if found, would represent the dynamical transition n_{dyn} . Our work in this paper suggests that this should set in well before the static crossover length n_{st} , for typical world languages.

Acknowledgments

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Appendix A. Chains with intra-cluster interactions

This appendix is devoted to the evaluation of the partition function Z_n of a binary chain of length n with arbitrary intra-cluster interactions. This quantity enters the definition (31) of the effective dynamical exponent Δ_n .

We consider the setting of the site percolation problem (see e.g. [38]). Most notations are chosen for the sake of consistency with the body of the paper. Each site is either occupied with probability q , or empty with the complementary probability $p = 1 - q$. Furthermore, arbitrary statistical weights λ_k are attached to each cluster of $k \geq 1$ consecutive occupied sites:

$$\circ \underbrace{\bullet \bullet \bullet}_{\lambda_2} \circ \circ \circ \underbrace{\bullet}_{\lambda_1} \circ \underbrace{\bullet \bullet \bullet}_{\lambda_3} \circ \underbrace{\bullet \bullet}_{\lambda_2} \circ \circ \underbrace{\bullet \bullet \bullet \bullet}_{\lambda_4} \circ \circ \quad (\text{A.1})$$

The total weight of a configuration is the product of the weights λ_k over all clusters. The partition function Z_n is the sum of these total weights over the 2^n configurations of an open finite chain of n sites, where each configuration is attributed a probability stemming from the site percolation problem. The partition function can be evaluated by means of recursion relations, somewhat along the lines of the transfer-matrix formalism (see e.g. [4, 9]). In the present situation, it is useful to write

$$Z_n = Z_n^\circ + Z_n^\bullet, \quad (\text{A.2})$$

where Z_n° (resp. Z_n^\bullet) are the partial partition sums over configurations whose leftmost site is empty (resp. occupied). The latter quantities obey the recursion relations

$$Z_n^\circ = p^n + \sum_{m=1}^{n-1} p^m Z_{n-m}^\bullet, \quad Z_n^\bullet = \lambda_n q^n + \sum_{m=1}^{n-1} \lambda_m q^m Z_{n-m}^\circ. \quad (\text{A.3})$$

The above equations are self-explanatory. The summation index is nothing but the size $m = 1, \dots, n - 1$ of the leftmost cluster of empty sites in the first equation,

of occupied sites in the second one. These recursion equations can be solved by introducing the generating series

$$G^\circ(z) = \sum_{n \geq 1} Z_n^\circ z^n, \quad G^\bullet(z) = \sum_{n \geq 1} Z_n^\bullet z^n, \quad (\text{A.4})$$

which obey

$$G^\circ(z) = \frac{pz}{1-pz}(1 + G^\bullet(z)), \quad G^\bullet(z) = L(z)(1 + G^\circ(z)), \quad (\text{A.5})$$

with

$$L(z) = \sum_{k \geq 1} \lambda_k q^k z^k. \quad (\text{A.6})$$

We thus obtain

$$G^\circ(z) = \frac{pz(1 + L(z))}{1 - pz(1 + L(z))}, \quad G^\bullet(z) = \frac{L(z)}{1 - pz(1 + L(z))}, \quad (\text{A.7})$$

and finally

$$G(z) = \sum_{n \geq 1} Z_n z^n = G^\circ(z) + G^\bullet(z) = \frac{pz + (1 + pz)L(z)}{1 - pz(1 + L(z))}. \quad (\text{A.8})$$

We are mostly interested in the asymptotic growth of the partition function Z_n for large chain lengths n . This growth law obeys the following dichotomy:

- If the cluster weights λ_k are bounded by an exponential of k , the generating series $L(z)$ has a non-zero radius of convergence. As a consequence, the partition function Z_n grows exponentially with the chain size n . This is in agreement with the existence of a well-defined thermodynamic limit where the total free energy is extensive, i.e.,

$$\ln Z_n \approx nK. \quad (\text{A.9})$$

The free energy density K per site is given by the property that

$$z_* = e^{-K} \quad (\text{A.10})$$

is the smallest zero of the denominator of (A.8).

It is worth considering a few examples.

For exponential cluster weights $\lambda_k = \mu^k$, introducing these weights boils down to the renormalisation of q into the product μq . We accordingly obtain the simple result

$$K = \ln(p + \mu q). \quad (\text{A.11})$$

For linear cluster weights $\lambda_k = ak$, we obtain a cubic equation for z_* :

$$(1 - pz_*)(1 - qz_*)^2 = apqz_*^2. \quad (\text{A.12})$$

For quadratic cluster weights $\lambda_k = bk^2$, we obtain a quartic equation:

$$(1 - pz_*)(1 - qz_*)^3 = bpqz_*^2(1 + qz_*). \quad (\text{A.13})$$

The two latter examples demonstrate that the expression of the free energy soon becomes quite intricate.

- If the cluster weights λ_k grow faster than any exponential, the generating series $L(z)$ has a vanishing radius of convergence. In such a circumstance, the partition function is dominated by the configuration where all sites are occupied, i.e.,

$$Z_n \approx q^n \lambda_n. \quad (\text{A.14})$$

The first correction to this leading result originates in the two configurations where only the first (or last) site is empty, yielding

$$Z_n = q^n \lambda_n \left(1 + \frac{2p\lambda_{n-1}}{q\lambda_n} + \dots \right). \quad (\text{A.15})$$

The correction term is negligible for large chain sizes, precisely because the cluster weights have a superexponential growth. The result (A.14) can be recast as

$$\ln Z_n = nK_n, \quad (\text{A.16})$$

where the effective free energy is superextensive, as its density

$$K_n \approx \ln q + \frac{\ln \lambda_n}{n} \quad (\text{A.17})$$

keeps growing indefinitely with the chain size n .

The situation where cluster weights grow factorially, according to

$$\lambda_k = \mu^k k!, \quad (\text{A.18})$$

plays a key role in the body of this paper (see (29)). There, the asymptotic growth of the above cluster weights is only marginally superexponential, as the corresponding free energy density exhibits a logarithmic growth, according to

$$K_n \approx \ln \frac{\mu q n}{e}. \quad (\text{A.19})$$

This is illustrated in Figure 4, showing plots of the free energy density K_n against n for several values of the parameter μ .

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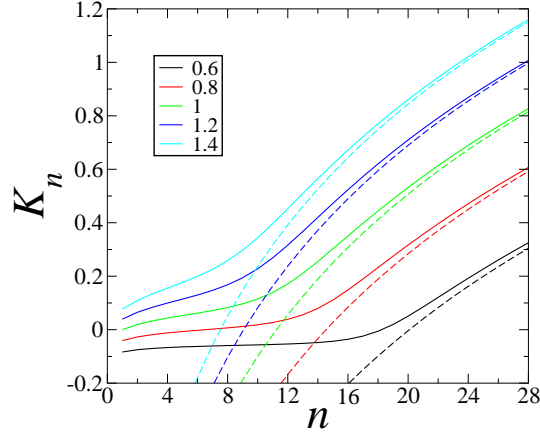


Fig. 4. Free energy density K_n against chain size n , for $q = 0.2$ and factorially growing cluster weights (see (A.18)), with several values of the parameter μ (see legend). Dashed curves: logarithmic asymptotic growth law (A.19).

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