Engineering Science
Mathematics Revision

August 2017 Prof DW Murray

Preamble

Throughout your time as an engineering student at Oxford you will receive lectures and tuition in the range of applied mathematical tools that today's professional engineer needs at her or his fingertips. The "1P1 series" of lectures starts in the first term with courses in Calculus and Series, Vectors and Matrices, Complex Algebra, and Differential Equations. Many of the topics will be familiar, others less so, but inevitably the pace of teaching and its style involving lectures and tutorials, will be wholly new to you.

To ease your transition, this introductory sheet provides a number of revision exercises related to these courses. Some questions may require you to read around. Although a few texts are mentioned on the next page, the material will be found in Further Maths A-level textbooks, so do not rush immediately to buy.

This sheet has not been designed to be completed in an evening, nor are all the questions easy. Including revision, and proper laying out of your solutions, the sheet probably represents up to a week's work. We suggest that you start the sheet at least three weeks before you come up so that your revision has time to sink in. The questions and answers should be still fresh in your mind by 1st week of term, when your college tutors are likely to review your work.

☞ **Do remember to bring your solutions to Oxford with you.**

What does "proper" laying out of your solutions mean? In the more involved questions it requires you not merely to slap the answer down, but to show and briefly explain the logical progression of your solution. Almost always in engineering questions and often in mathematical questions (e.g. when using vectors) it is useful to sketch a labelled diagram as part of your solution.

Reading

The ability to learn new material yourself is an important skill which you must acquire. But, like all books, mathematics for engineering texts are personal things. Some like the bald equations, others like to be given plenty of physical insight. However, three useful texts are:

Title:	Advanced Engineering Mathematics
Author:	E. Kreyszig
Publisher:	John Wiley & Sons
Edition:	10th ed. (2011)
ISBN:	9780470646137 (Paperback c.£60 new)

Title:	Advanced Engineering Mathematics
Author:	K. Stroud (with D. J. Booth)
Publisher:	Palgrave Macmillan
Edition:	7th ed. (2013)
ISBN:	978-1-137-03120-4 (Paperback c.£45 new)

Title:	Mathematical Methods for Science Students
Author:	G Stephenson
Publisher:	Longman
Edition:	2nd ed. (1973)
ISBN/ISSN:	0582444160 (Paperback c.£53 new – ouch!)

Kreyszig's book is quite comprehensive and will be useful throughout your course and beyond. Stroud's text covers material for the 1st year and is well reviewed by students. Stephenson's book again covers 1st year material, but is divorced from engineering applications. It is packed with examples, but it is a bit dull.

As mentioned earlier, don't rush to buy these. But if you want to, think paperback and second-hand rather than new, and please shop around. In particular, avoid new copies of Stephenson: this old warhorse seems overpriced.

1. Differentiation

☞ You should be able to differentiate simple functions:

1. $5x^2$

2. $4\tan x$

3. $4e^x$

4. $\sqrt{1+x}$

☞ use the chain rule to differentiate more complicated functions:

5. $6\cos(x^2)$

6. e^{3x^4}

☞ know the rules for differentiation of products and quotients:

7. $x^2 \sin x$

8. $\dfrac{\tan x}{x}$

☞ understand the physical meaning of the process of differentiation:

9. The velocity of a particle is given by $20t^2 - 400e^{-t}$. Determine its acceleration at time $t = 2$.

10. Find the stationary points of the function $y = x^2 e^{-x}$, and determine whether each such point is a maximum or minimum.

2. Integration

☞ You should understand the difference between a definite and an indefinite integral, and be able to integrate simple functions by recognising them as derivatives of familiar functions:

11. $\int_a^b 3x^2 dx$

12. $\int (x^4 + x^3) dx$

13. $\int \sin x \cos^5 x dx$

14. $\displaystyle\int \frac{x}{\sqrt{1-x^2}} dx$

☞ be able to manipulate functions so that more complex functions become recognisable for integration:

15. $\int_0^{2\pi} \sin^2 x \, \mathrm{d}x$

16. $\int \tan x \, \mathrm{d}x$

☞ change variables, e.g. using $x = \sin\theta$ or some other trigonometric expression, to integrate functions such as:

17. $\displaystyle\int \frac{1}{\sqrt{1-x^2}} \, \mathrm{d}x$

18. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}} \, \mathrm{d}x$

☞ use integration by parts for certain more complicated functions:

19. $\int x \sin x \, \mathrm{d}x$

☞ understand the physical meaning of integration:

20. What is the area between the curve $y = 8x - x^4$ and the x-axis for the section of the curve starting at the origin which lies above the x-axis?

21. The velocity of a particle is $20t^2 - 400\mathrm{e}^{-t}$, and the particle is at the origin at time $t = 0$. Determine how far it is from the origin at time $t = 2$.

3. Series

☞ You should be able to sum arithmetic and geometric series:

22. Sum (using a formula, not by explicit addition!) the first ten numbers in the series

$$10.0, \quad 11.1, \quad 12.2, \quad \ldots$$

23. Sum the first ten terms of the series

$$x, \quad 2x^2, \quad 4x^3, \quad \ldots$$

☞ understand what a binomial series is:

24. Find the first four terms in the expansion of $(a + 2x)^n$, where n is an integer and $n > 3$.

4. Functions

☞ You should be familiar with the properties of standard functions, such polynomials, rational functions (where both numerator and denominator are polynomials), exponential functions, logarithmic functions, and trigonometric functions and their identities:

25. i) For what value(s) of x is the function $f(x) = x/(x^2 - 1)$ undefined? Describe the behaviour of f as x approaches these values from above and below.

ii) Find the limits of $f(x)$ and df/dx as $x \to +\infty$ and $x \to -\infty$.

iii) Does the function have stationary values? If so, find the values of x and $f(x)$ at them.

iv) Now make a sketch of the function, labelling all salient features.

26. Sketch $y = e^{-t}$ and $y = e^{-3t}$ versus time t for $0 < t < 3$. When a quantity varies as $e^{-t/\tau}$, τ is called the *time constant*. What are the time constants of your two plots? Add to your sketch two curves showing the variation of a quantity with (i) a very short time constant, (ii) a very long time constant.

27. A quantity varies as $y = 100e^{-10t} + e^{-t/10}$. Which part controls the behaviour of y at short time scales (ie when t is just above zero), and which at long times-scales?

28. A quantity y_1 varies with time t as $y_1 = 2\cos\omega t$. A second quantity y_2 varies as $y_2 = \cos(2\omega t + \frac{\pi}{4})$. Plot y_1 and y_2 versus ωt, for $-2\pi < \omega t < 2\pi$. What are the amplitudes and frequencies of y_1 and y_2?

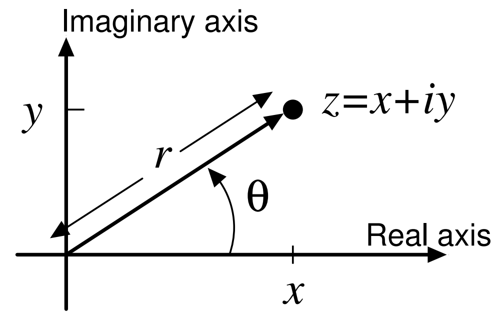
☞ The hyperbolic cosine is defined as $\cosh x = \frac{1}{2}(e^x + e^{-x})$, and the hyperbolic sine is defined as $\sinh x = \frac{1}{2}(e^x - e^{-x})$. Other hyperbolic functions are defined by analogy with trigonometric functions: eg, the hyperbolic tangent is $\tanh x = \sinh x/\cosh x$.

29. Show that
(i) $\cosh^2 x - \sinh^2 x = 1$; (ii) $(1 - \tanh^2 x)\sinh 2x = 2\tanh x$.

30. Find $\frac{\mathrm{d}}{\mathrm{d}x}\cosh x$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sinh x$. (Express your results as hyperbolic functions.)

5. Complex Algebra

☞ You should find this topic in most A-level texts. We will use the notation that a complex number $z = (x + iy)$, where x and y are the **Real** and **Imaginary** parts of z, respectively. That is, $x = \mathrm{Re}(z)$ and $y = \mathrm{Im}(z)$. The **Imaginary unit** i is such that $i^2 = -1$. Complex numbers can be represented as points on an Argand diagram. The modulus or magnitude of the complex number is r, where $r^2 = x^2 + y^2$, and the argument is θ. Obviously $x = r\cos\theta$, and $y = r\sin\theta$.



31. Evaluate (i) $(1 + 2i) + (2 + 3i)$; (ii) $(1 + 2i)(2 + 3i)$; (iii) $(1 + 2i)^3$ and plot the resulting complex numbers on an Argand diagram.

32. If $z = (x + iy)$, its **complex conjugate** is defined as $\overline{z} = (x - iy)$.
Show that $z\overline{z} = (x^2 + y^2)$.

33. By multiplying top and bottom of the complex fraction by the complex conjugate of $(3 + 4i)$, evaluate $\dfrac{1 + 2i}{3 + 4i}$.

34. Using the usual quadratic formula, find the two complex roots of $z^2 + 2z + 2 = 0$. (Hint: as $i^2 = -1$ we have that $\sqrt{-1} = \pm i$.) Are complex solutions to a quadratic equation always conjugates?

35. Using standard trigonometrical identities, show that

$$(\cos\theta + i\sin\theta)^2 = (\cos 2\theta + i\sin 2\theta).$$

(More generally, $(\cos\theta + i\sin\theta)^\alpha = (\cos\alpha\theta + i\sin\alpha\theta)$ for *any* α.)

6. Vectors

Below, vectors are written in bold, unit vectors in the (x, y, z) directions are $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ and a vector from point A to point B may be written $\overrightarrow{AB}$.

☞ You should be familiar with the vector algebra of points, lines and planes, and with the scalar product.

36. Find the **unit** vector $\hat{\mathbf{v}}$ in the direction $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

37. Find the coordinates of point P if $|\overrightarrow{OP}| = 3$ and vector $\overrightarrow{OP}$ is in the direction of (i) $\mathbf{i}+\mathbf{j}+\mathbf{k}$, (ii) $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$. ($O$ is the origin.)

38. Write down the vector equation of the straight lines (i) parallel to $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and through the origin, (ii) parallel to $\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and through the point (1,1,1).

39. Find the point on the line $\mathbf{i}+\mathbf{j}+\mathbf{k}$ that is nearest to the point $(3, 4, 5)$.

40. Determine the angle between the vectors $(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ and $(3\mathbf{i}+2\mathbf{j}+\mathbf{k})$.

41. Find the vector position of a point 1/3 of the way along the line between (x_1, y_1, z_1) and (x_2, y_2, z_2), and nearer (x_1, y_1, z_1).

42. At time $t = 0$ two forces $\mathbf{f}_1 = (\mathbf{i}+\mathbf{j})$ and $\mathbf{f}_2 = (2\mathbf{i}-2\mathbf{j})$ start to act on a point body of unit mass which lies stationary at point $(1, 2)$ of the x, y plane. Determing the subsequent trajectory $\mathbf{r}(t)$ of the particle.

Answers and hints

1. $10x$

2. $4\sec^2 x$

3. $4e^x$

4. $1/(2\sqrt{1+x})$

5. $-12x\sin(x^2)$

6. $12x^3 e^{3x^4}$

7. $x^2\cos x + 2x\sin x$

8. $(x\sec^2 x - \tan x)/x^2$

9. $80 + 400/e^2 \approx 134.1$

10. Min at $(0, 0)$, Max at $(2, 4e^{-2})$

11. $b^3 - a^3$

12. $x^5/5 + x^4/4 + C$

13. $-\frac{1}{6}\cos^6 x + C$

14. $-\sqrt{1-x^2} + C$

15. π

16. $-\ln(\cos x) + C$, where ln denotes $\log_e$

17. $\sin^{-1} x + C$

18. $\sin^{-1}(x/a) + C$

19. $-x\cos x + \sin x + C$

20. 9.6

21. $(160/3) + (400/e^2) - 400 \approx -292.5$

22. 149.5

23. $\dfrac{x(1-1024x^{10})}{1-2x}$

24. $a^n + 2na^{n-1}x + 2n(n-1)a^{n-2}x^2 + \frac{4}{3}n(n-1)(n-2)a^{n-3}x^3 + \ldots$

25. (i) $f(x) = x/(x^2-1)$ undefined at $x = \pm 1$. Asymptotic behaviour at $x = \pm 1$. (ii) As $x \to +\infty$, $f(x) \to 0$ from above. As $x \to -\infty$, $f(x) \to 0$ from below. Gradients both tend to zero. (iii) $\mathrm{d}f/\mathrm{d}x = -(x^2+1)/(x^2-1)^2$ is nowhere zero, hence no turning points.

26. $y = e^{-t}$ and $y = e^{-3t}$: time constants 1 and 1/3 respectively.

27. $100e^{-10t}$ dominates at small t. Note cross over when $100e^{-10t} = e^{-t/10}$ or $e^{-9.9t} = 0.01$, ie at $t = 0.46$.

28. Amplitude 2, frequency $f = \omega/2\pi$;
Amplitude 1, frequency $f = \omega/\pi$.

† Please see the suggestion at the bottom of the page.

29. (i) $\cosh^2 x = (e^{2x} + e^{-2x} + 2)/4$;
$\sinh^2 x = (e^{2x} + e^{-2x} - 2)/4$;
$\cosh^2 - \sinh^2 = 4/4 = 1$.

(ii) $1 - \tanh^2 = 1/\cosh^2$;
$\sinh 2x = 2\cosh x \sinh x$; Hence
$(1 - \tanh^2 x)\sinh 2x =$
$2\cosh x \sinh x / \cosh^2 x = 2\tanh x$.

30. $\frac{d}{dx}(e^x + e^{-x})/2 = (e^x - e^{-x})/2$. Hence $\frac{d}{dx}\cosh x = \sinh x$ and similarly $\frac{d}{dx}\sinh x = \cosh x$.

31. (i) $(3 + 5i)$; (ii) $(-4 + 7i)$; (iii) $(-11 - 2i)$;

32. Note $\sqrt{x^2 + y^2}$ is the *modulus* of z (and of $\overline{z}$ too for that matter).

33. (11/25) + i(2/25)

34. Solutions are $(-1 \pm i)$. Yes: for a complex soln. The usual formula gives roots as

$$(-b \pm \sqrt{b^2 - 4ac})/2a.$$

For complex roots, $b^2 - 4ac < 0$, giving the imaginary part and $\pm$ signs always gives conjugate pairs with the same real part. Note though if $b^2 - 4ac > 0$ the two real solutions are different.

35. (i) Square to find $(cos^2\theta - \sin^2\theta + 2i\sin\theta\cos\theta)$, hence result.

36. $\hat{\mathbf{v}} = \frac{1}{\sqrt{6}}(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.

37. (i) $(\sqrt{3}, \sqrt{3}, \sqrt{3})$, (ii) $\frac{3}{\sqrt{14}}(1, -2, 3)$.

38. (i) $\mathbf{r} = \dfrac{\alpha}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$, where parameter α is any real number. (NB: strictly no need for the $\sqrt{3}$, but using it makes α a measure of distance).

(ii)

$$\mathbf{r} = \left(1 + \frac{\alpha}{\sqrt{6}}\right)\mathbf{i} + \left(1 - \frac{2\alpha}{\sqrt{6}}\right)\mathbf{j} + \left(1 + \frac{\alpha}{\sqrt{6}}\right)\mathbf{k}$$

(Again no real need for $\sqrt{6}$, but ...)

39. Vector from point to a general point on line is $(\frac{\alpha}{\sqrt{3}} - 3)\mathbf{i} + (\frac{\alpha}{\sqrt{3}} - 4)\mathbf{j} + (\frac{\alpha}{\sqrt{3}} - 5)\mathbf{k}$. We want α corresponding to minimum distance, or minimum squared-distance. Squared distance is $d^2 = (\frac{\alpha}{\sqrt{3}} - 3)^2 + (\frac{\alpha}{\sqrt{3}} - 4)^2 + (\frac{\alpha}{\sqrt{3}} - 5)^2$.. Diff wrt α and set to zero, cancelling factor of $2/\sqrt{3}$, gives $(\frac{\alpha}{\sqrt{3}} - 3) + (\frac{\alpha}{\sqrt{3}} - 4) + \frac{\alpha}{\sqrt{3}} - 5) = 0$, so that $\alpha = 4\sqrt{3}$. Thus the closest point is $(4, 4, 4)$.

40. Take the scalar product of **UNIT** vectors! $\cos^{-1}(10/14) = 44.41°$.

41. $(x_1, y_1, z_1) + \frac{1}{3}[(x_2, y_2, z_2) - (x_1, y_1, z_1)] = \frac{1}{3}[(2x_1 + x_2), (2y_1 + y_2), (2z_1 + z_2)]$.

42. Total force is $(3\mathbf{i} - \mathbf{j})$ so for unit mass, $\ddot{x} = 3; \ddot{y} = -1$. Thus $\dot{x} = 3t + a; \dot{y} = -t + b$ where $a = b = 0$, as stationary at $t = 0$. Hence $x = 3t^2/2 + c$ and $y = -t^2/2 + d$, where, using initial position, $c = 1$ and $d = 2$.

Finally

$$\mathbf{r}(t) = (3t^2/2 + 1)\mathbf{i} + (-t^2/2 + 2)\mathbf{j}\ .$$

† To check your plot you could visit www.wolframalpha.com and type this into the box

plot y=2cos(x) and y=cos(2x+pi/4) for -2pi<x<2pi

Revision 2

Part A Statics and Dynamics

Introduction

The questions in this short introductory examples sheet deal with material which is mainly covered in A Level Physics or Mathematics. They are intended to help you make the transition between school work and the Engineering Science course at Oxford. You should attempt these questions before you come to Oxford and be prepared to discuss any difficulties with your tutor when you first meet with them. The P3 Statics lectures, which will take place at the beginning of your first term, will build on the topics covered in the first group of problems. Although the P3 Dynamics lectures will not take place until later in the academic year, it is still essential for you to attempt the second group of problems at this stage.

For the acceleration due to gravity use g = 10 m/s^2.

Statics Problems

1. An aeroplane with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine 3 suddenly fails. The relevant dimensions are shown in Figure 1. Determine the resultant of the three remaining thrust forces, and its line of action.

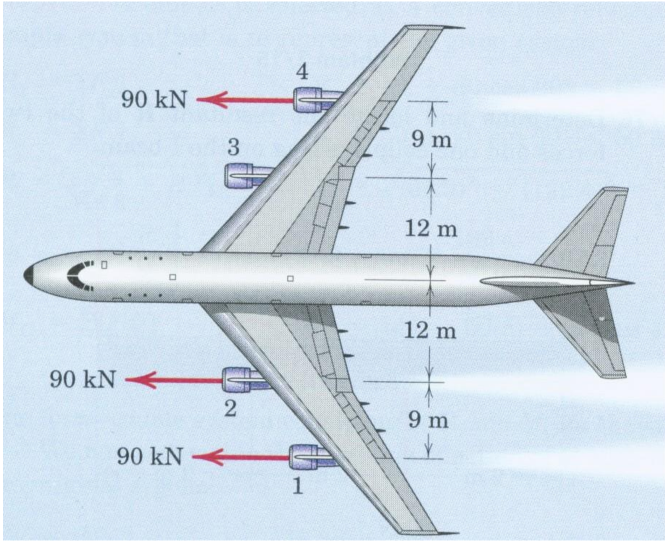


Figure 1

2. The foot of a uniform ladder rests on rough horizontal ground while the top rests against a smooth vertical wall. The mass of the ladder is 40 kg. A person of mass 80 kg stands on the ladder one quarter of its length from the bottom. If the inclination of the ladder is 60° to the horizontal, calculate:

a) the reactions at the wall and the ground;

b) the minimum value of the coefficient of friction between the ground and the ladder to prevent the ladder slipping.

3. Figure 2 shows a tower crane. The counterweight of 1500 kg is centred 6 m from the centreline of the tower. The distance *x* of the payload from the centreline of the tower can vary from 4 to 18 m.

a) Calculate the moment reaction at the base of the tower with:

- no payload
- payload of 1000 kg at x = 4 m
- payload of 1000 kg at x = 18 m

b) Show that the effect of the counterweight is to reduce the magnitude of the maximum moment reaction by a factor of 2.

c) Explain why changing the size of the counterweight would be detrimental.

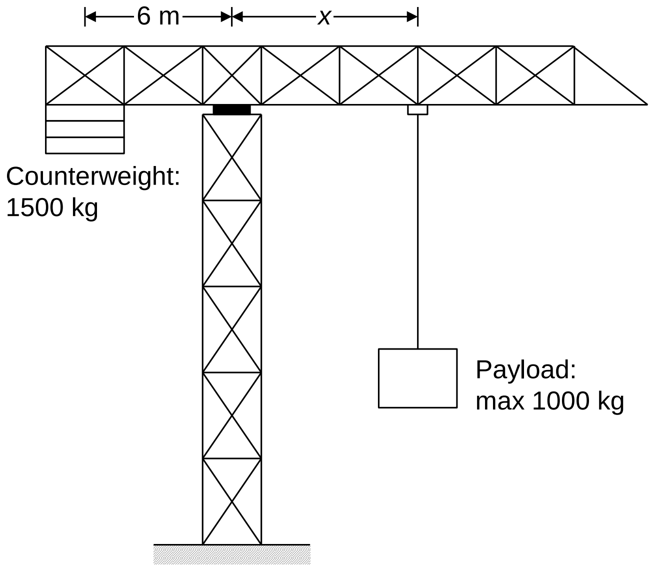


Figure 2

4. a) Figure 3 shows Galileo's illustration of a cantilever (i.e. a beam that is rigidly fixed at one end and unsupported at the other). If the beam is 2 m long and has mass per unit length of 7.5 kg/m, and the rock E has mass 50 kg, calculate the vertical reaction and the moment reaction at the wall.

b) A second cantilever tapers so that its mass per unit length varies linearly from 10 to 5 kg/m from the left hand to right hand ends, and it does not carry a rock at its free end. Calculate the vertical and moment reactions at the wall.

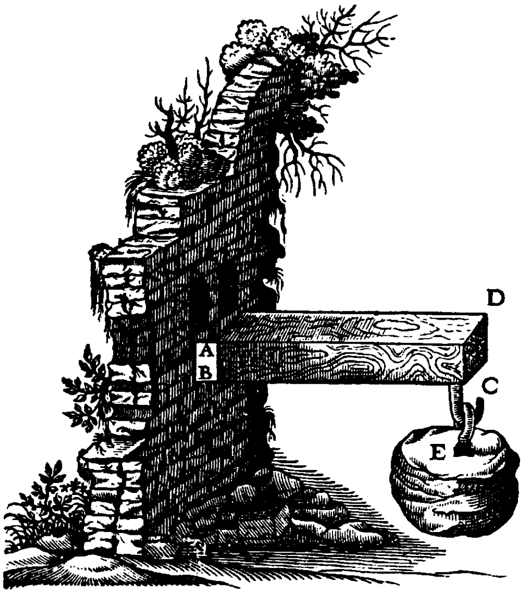


Figure 3

5. Figure 4 shows a plan view of a circular table of radius 400 mm and weight 400 N supported symmetrically by three vertical legs at points A, B and C located at the corners of an equilateral triangle of side 500 mm. An object weighing 230 N is placed at a point D on the bisector of angle ABC and a distance x from AC. Assume that the reactions at A, B and C are vertical.

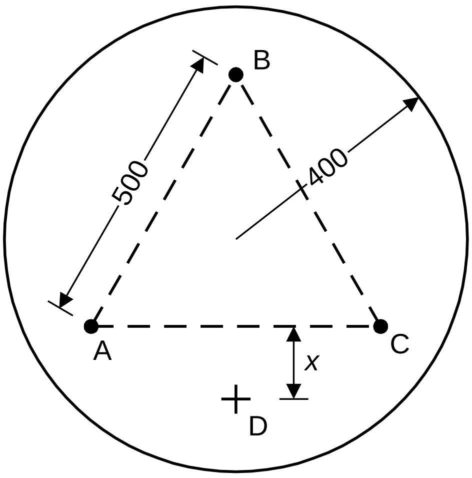


Figure 4

a) Find the value of *x* and the values of the reactions at which the table starts to tip.

b) Explain why the table cannot tip if the object weighs 220 N.

6. Blocks A and B have mass 200 kg and 100 kg respectively and rest on a plane inclined at 30° as shown in Figure 5. The blocks are attached by cords to a bar which is pinned at its base and held perpendicular to the plane by a force *P* acting parallel to the plane.Assume that all surfaces are smooth and that the cords are parallel to the plane.

a) Draw a diagram of the bar showing all the forces acting on it.

b) Calculate the value of *P*.

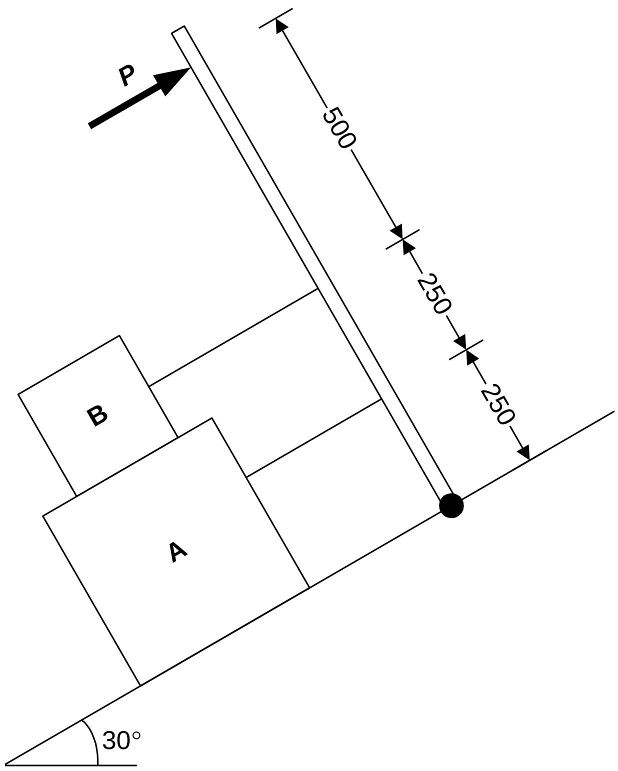


Figure 5

7. Figure 6 shows a uniform bar of weight W suspended from three wires. An additional load of $2W$ is applied to the bar at the point shown.

a) Draw a diagram of the bar showing all the forces acting on it.

b) Write down any relevant equilibrium equations and explain why it is not possible to calculate the tensions in the wires without further information.

c) In one such structure it is found that the centre wire has zero tension. Calculate the tensions in the other two wires.

d) In a second such structure assume that the wires are extensible and the bar is rigid. Write down an expression for the extension of the middle wire in terms of the extensions of the two outside wires. Assuming the tensions in the wires are proportional to the extensions, calculate the tensions for this case.

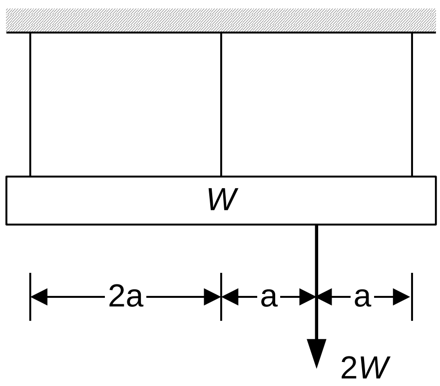


Figure 6

8. The three bars in Figure 7 each have a weight W. They are pinned together at the corners to form an equilateral triangle and suspended from A.

a) Draw a diagram of each bar separately, showing all the forces acting on each bar.

b) Calculate the compressive force in bar BC.

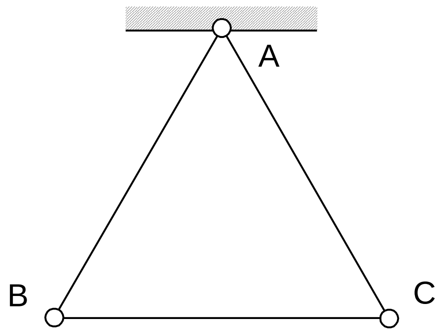


Figure 7

Dynamics Problems

9. A body with initial velocity u has constant acceleration a. Starting from the definition that velocity is the rate of change of displacement, and acceleration is the rate of change of velocity, show that:

a) $v = u + at$

b) $v^2 = u^2 + 2as$

c) $s = ut + \frac{1}{2}at^2$

d) A stone takes 4 s to fall to the bottom of a well. How deep is the well? What is the final velocity of the stone? What problems would you encounter in this calculation if the stone took 50 s to reach the bottom?

e) How do the equations in a), b) and c) change if the acceleration is not constant?

10. A car engine produces power of 20 kW. If all of this power can be transferred to the wheels and the car has a mass of 800 kg, calculate:

a) the speed which the car can reach from rest in 7 s;

b) the acceleration at time 7 s.

Is it reasonable to assume the power is constant? How does the gearbox in the car help to make this a more reasonable assumption?

11. A stone of mass m is tied on the end of a piece of string. A child swings the stone around so that it travels in a horizontal circle of radius r at constant angular velocity ω rad/s. Write down expressions for:

a) the speed of the stone;

b) the time to travel once around the circle;

c) the acceleration of the stone, specifying its direction;

d) the kinetic energy of the stone;

e) the tension in the string and the angle it makes with the horizontal if the gravitational acceleration is g.

12. a) A bicycle wheel has radius R and mass m, all of which is concentrated in the rim. The spindle is fixed and the wheel rotates with angular velocity ω. Calculate the total kinetic energy of the wheel. How does the kinetic energy differ from this if the wheel is rolling along with angular velocity ω, rather than spinning about a fixed axis?

b) In contrast to part a), a disc of mass m, radius R and angular velocity ω has its mass uniformly distributed over its area. Calculate the total kinetic energy of the disc as follows:

i) Write down the mass of the disc contained between radius r and radius $r + dr$.

ii) Write down the speed of this mass.

iii) Calculate the kinetic energy of this mass.

iv) Calculate the total kinetic energy of the whole disc by integrating the previous result with respect to r between the limits $r = 0$ and $r = R$.

Answers

1. 270 kN, 4 m from centreline

2. a) $H_{wall} = 400/\sqrt{3}\,\text{N}$, $H_{ground} = 400/\sqrt{3}\,\text{N}$, $V_{ground} = 1200\,\text{N}$

b) $\mu \geq 1/\left(3\sqrt{3}\right)$

3. a) M = -90, -50, +90 kNm (+ve = anti-clockwise)

4. a) 650 N, 1150 Nm

b) 150 N, 133.3 Nm

5. a) $R_A = R_C = 315\,\text{N}$, $R_B = 0\,\text{N}$, $x = 251\text{mm}$

6. b) $P = 500\,\text{N}$

7. c) W, $2W$

d) $W/2$, W, $3W/2$

8. b) $W/\sqrt{3}$

9. d) 80 m, 40 m/s

10. a) 18.7 m/s

b) 1.34 m/s^2

11. a) $r\omega$

b) $2\pi/\omega$,

c) $r\omega^2$

d) $\frac{1}{2}mr^2\omega^2$

e) $\theta = \tan^{-1}\left(g/r\omega^2\right)$, $T = m\sqrt{g^2 + r^2\omega^4}$

12. a) $\frac{1}{2}mR^2\omega^2$, or $mR^2\omega^2$ if rolling

b) $\frac{1}{4}mR^2\omega^2$

Revision 2

Part B Electricity

Instructions

Do as much as you can **before** you come up to Oxford. Most of the questions are based on material that you should have covered in A level physics. All the topics will be covered in the initial lectures (and tutorials) at Oxford but you should consult books if you are stuck. The recommended text is "Electrical and Electronic Technology" by Hughes *et al* published by Pearson Higher Education/Longman, but many of the basic ideas can also be found in some A level texts. Some numerical answers are given at the end.

Basic concepts

1. Current as a flow of charge (Hughes 2.4 Movement of electrons)

   A metal wire 1m long and 1.2 mm diameter carries a current of 10 A. There are 10^{29} free electrons per m^3 of the material, and the electron charge is 1.6 x 10^{-19} C. On average, how long does it take an electron to travel the whole length of the wire?

2. Resistance and resistivity (*Hughes 3.5 and 3.6: Power and energy, Resistivity*)

   An electromagnet has a coil of wire with 1400 turns, in 14 layers. The inside layer has a diameter of 72 mm and the outside layer has a diameter of 114 mm. The wire has a diameter of 1.6 mm and the resistivity of warm copper may be taken as 18 nΩm.

i) What is the approximate resistance of the coil?

ii) What is the approximate power dissipated as heat if the coil carries a current of 6 A?

[Hint: average turn length = π x average diameter]

3. A laminated conductor is made by depositing, alternately, layers of silver 10 nm thick and layers of tin 20 nm thick. The composite material, considered on a larger scale, may be considered a homogeneous but anisotropic material with electrical resistivity $\rho_\perp$ for currents perpendicular to the planes of the layers, and a different resistivity, ρ_p for currents parallel to that plane. Given that the resistivity of tin is 7.2 times that of silver find the ratio of the resistivities, $\rho_\perp / \rho_p$.

Engineering models

To analyse real physical systems, engineers have to describe their components in simple terms. In circuit analysis the description usually relates voltage and current – for example through the concept of resistance and Ohm’s law. Often an engineer has to make assumptions to simplify the analysis, and must ensure that these assumptions are justified.

4. The ideal conductor

What are the properties of 'ideal' conductors used in circuit diagrams and how do they differ from real conductors?

5. The conductor in a circuit

A thin copper wire of radius 0.5 mm and total length 1 m, is used to connect a 12 V car battery to a 10 W bulb.

i) Estimate the resistances of the wire and the bulb.

ii) What would be a suitable model for the wire?

iii) What assumptions have you made? (Think about the battery, the wire and the bulb.)

iv) There is now a fault in the bulb, and it acts a short circuit. What model of the wire is now appropriate?

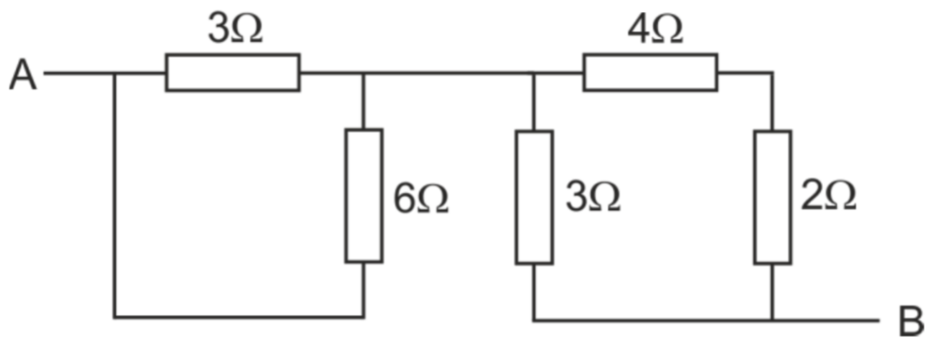
v) Do you now need to change any of your assumptions?

Circuit analysis

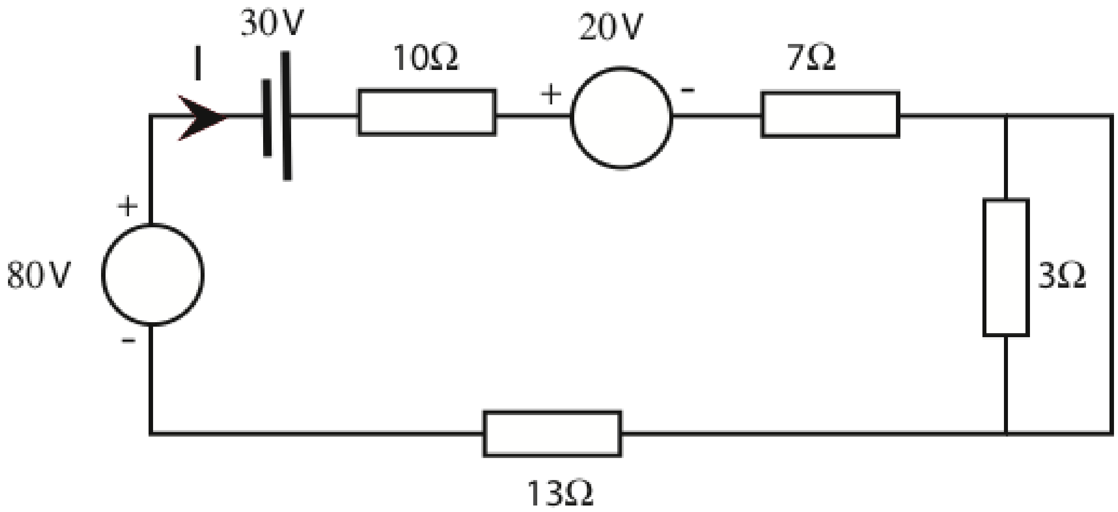
One of the fundamental techniques in electricity is circuit analysis. The algebra is usually easier if you work either with currents as unknowns or voltages. These are basically restatements of Ohm's Law.

6. The idea of resistance can be extended to describe several components together.

i) Find the resistance, R_{AB}, between A and B in the circuit below. (Hint: re-draw the circuit combining the series and parallel components. You need not do this in one step.)

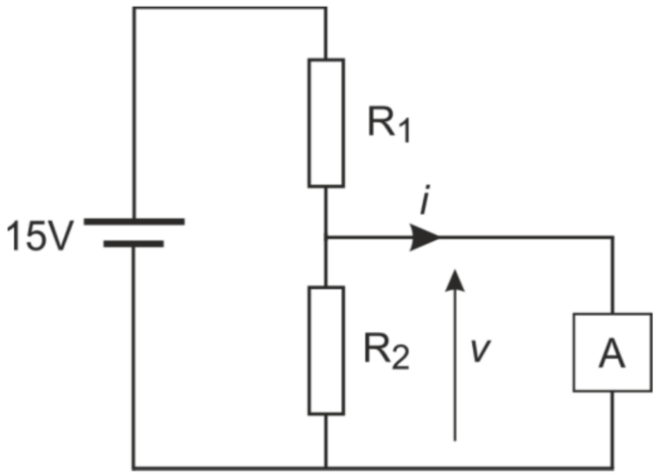


ii) Find the current, *I*, in the circuit below



iii) If an infinite number of resistors, not necessarily having the same value, are connected in series to what limit does the overall resistance of the combination tend? What is the limit if the resistors are now connected in parallel?

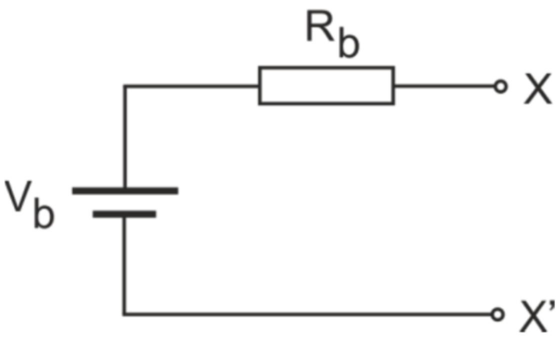
7. For the circuit shown, choose R_1 and R_2 so that the voltage v is 10 V when the device A takes zero current, but falls to 8 V when i rises to 1 mA.



Circuit models

8. The battery

A battery generates voltage through an electrochemical process; the voltage drops a little as the battery supplies more current. You have met the idea of modelling the battery by an ideal voltage source, V_b and resistance R_b (see the figure below, in which the battery **terminals** are at XX'). If the voltage with no current is 9 V but the terminal voltage drops to 8.8 V when a current of 1A is drawn from it, find the values of V_b and R_b.



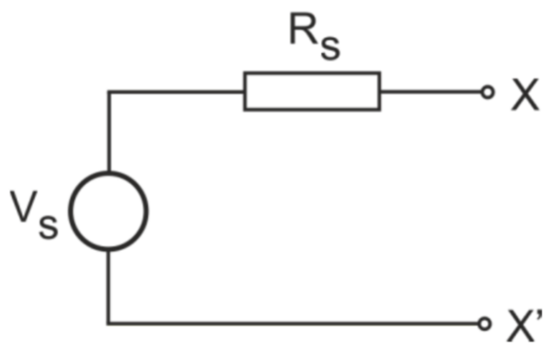
9. A general voltage source

This model can be extended to any voltage source. For example in the laboratory you will use voltage generators which supply, say, a sine wave. Inside these are a number of circuit components. However as far as the outside world is concerned they can be modelled in exactly the same way: as a voltage, V_S and a resistance. The resistance is often called the source resistance, R_S or the output resistance, $R_{out.}$ An example is shown in the figure below.

i) If V_S = 5V what is the voltage at XX' in the circuit as shown? This is called the *open circuit* voltage since there is an open circuit across the terminals.

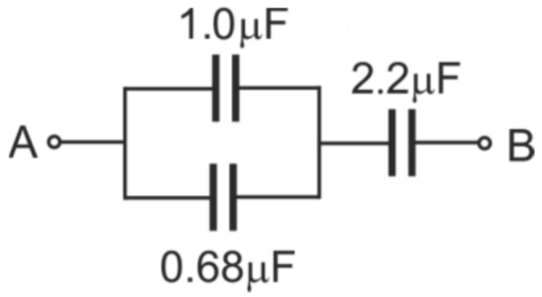
ii) What is the voltage across the terminals XX' if R_S is 10 Ω and a resistance of 100 Ω is connected across them?

iii) In another source generating the same voltage a resistance of 100 Ω across XX' results in a terminal voltage of 4.9V. What is R_S?



Capacitors and Inductors

10. What is the apparent capacitance between A and B?



11. Write down the definitions of resistance, capacitance and inductance in terms voltage, charge and current.

    i) If an a.c. voltage of $V_0 \sin(\omega t)$ is applied to each of the components, write down an expression for the *current* in each case. (Hint: Remember current is the rate of change of charge. You may need to look up the behaviour of an inductor.)

    ii) Remember that power in an electrical circuit is the product of voltage and current. In an a.c. circuit both voltage and current are varying with time (as in the calculation you have just done for the resistor, capacitor and inductor) so the power must also be varying with time. Write down an expression for the power for each of the three cases.

    iii) Now work out the average power in each case.

Some answers

1. About half an hour

2. 3.66 Ω 132 W

3. 2.19

5. 0.023 Ω, 14.4 Ω

6. 4 Ω

7. R_1 = 3 kΩ, R_2 = 6 kΩ

8. 9 V, 0.2 Ω

9. 5 V, 4.54 V, 2.04 V

10. 0.953 µF

11. ii) ${V_0}^2[1 - \cos(2\omega t)]/R,\ \omega C{V_0}^2\sin(2\omega t), -({V_0}^2/\omega\mathrm{L})\sin(2\omega t)$

    iii) ${V_0}^2/2\mathrm{R}$, 0, 0

Introduction to Computing

Computing is a central part of the professional engineer's working life. While at Oxford, you will come across professional engineering software packages for Computer Aided Design, Computational Fluid Dynamics and many other applications. A common software packaged, used by many engineers in industry and academia, is MATLAB. At its most fundamental level, it is like a programmable scientific calculator, but with the file and memory resources of a computer at its disposal. The aspect that sets MATLAB apart from other software packages is it's ability to efficiently carry out computations on large vectors and matrices.

Importantly MATLAB also contains its own programming language. This is important as there are many occasions when the software you need does not exist, so you will need to be able to program your own. The next pages introduce MATLAB and basic concepts in programming. We do not expect any prior knowledge. If this is your first introduction to programming, read the information and try to grasp the content. If you already have experience of coding, let these exercises be a refresher for you, and an introduction to MATLAB specific syntax and functions.

This introductory information ends with a short quiz you must attempt. While it is beneficial to have access to MATLAB to follow along with the content, it is not necessary. Please do not worry if you cannot access the software as described on the next page.

Accessing MATLAB

There are computer laboratories in the department where you will be taught how to use MATLAB and other software. The Department also pays for a MATLAB license so that you can install the software on your own computer for free. This means you are able to work using software MATLAB outside of scheduled laboratory time - particularly useful when have to use MATLAB for coursework projects. To access MATLAB using the university license, you will need access to your university e-mail address. Your must verify your student status by setting up a MathWorks account with your university e-mail address. The next page has instructions for various ways to access MATLAB.

Toolboxes When installing MATLAB you will be shown a checkbox menu where you can choose additional **toolboxes** to install. These expand the functions available in but can lead to large install size. Alone MATLAB is around 2GB, but toolboxes can easily make this 4-6GB. When you are just starting out using MATLAB you will not need this extra functionality yet. **It is recommended you only install MATLAB, the Symbolic Math Toolbox and SIMULINK**. Other toolboxes can be added later if you need them.

System Requirements You will need at least 2GB RAM, but 4GB is better. If you have concerns about your computers specification, detailed system requirements can be found at

`https://uk.mathworks.com/support/sysreq.html.`

If you have access to...

- **your university e-mail and a personal computer**:
  Instructions for installing MATLAB are in the READ ME FIRST file, found after selecting *Matlab Student Edition* at `https://register.oucs.ox.ac.uk/self/software`.
- **your university e-mail and a tablet or smartphone**:
  You can follow the instructions in the READ ME FIRST file, as accessed above. Once you have your activated MathWorks account you will be able to access a limited version of MATLAB remotely using the MATLAB App or in browser through MATLAB Online at `https://matlab.mathworks.com/`
- **your own computer but not your university e-mail yet**:
  You can optionally download a free 30-day trial from
  `https://uk.mathworks.com/`
  Scroll to the bottom of the webpage to see a download link. You will then be able to activate the license once you have access to a university account.

If you do not have access to MATLAB, do not worry.
Simply read through the notes.

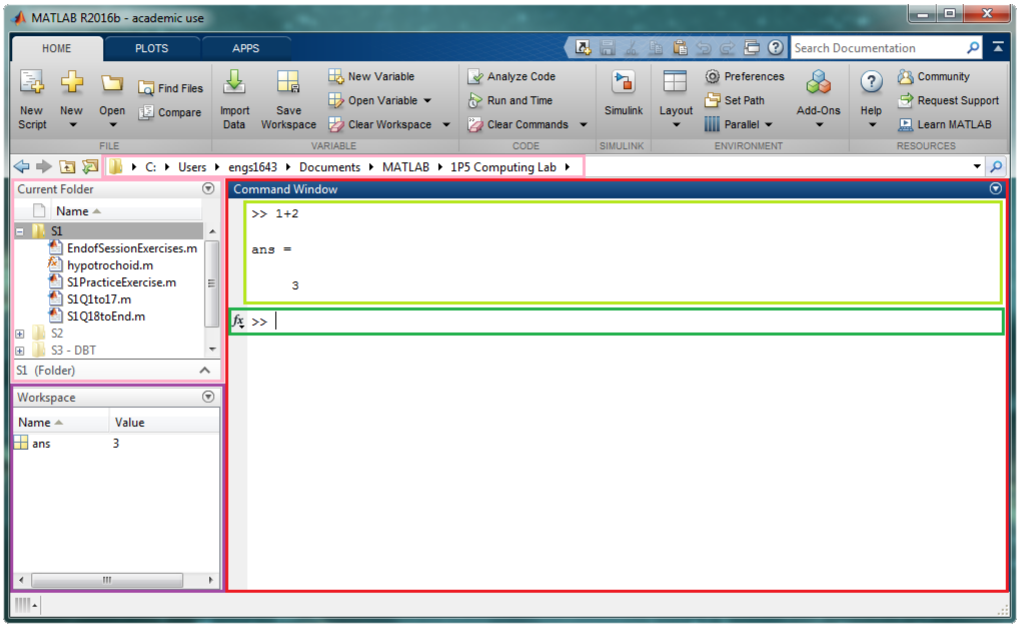


Figure 1: MATLAB 2016b Interface

Figure 1 shows the standard layout of MATLAB 2016b upon launch. Menus run across the top and various windows fill the rest of the screen. The three main windows are:

- the **Command Window** (outlined in red):

  Anything that can be done in MATLAB can be achieved by typing into the Command Window. The dark green area shows where you can type, and the light green shows previous commands that have been executed.

- the **Workspace** (outlined in purple):

  Shows any information that is currently stored in MATLAB's memory. In the Figure the value '3' is currently stored as 'ans'.

- the **Current Directory** (outlined in pink):

  Shows MATLAB's working directory, that is what files it has access to, and what folder any files created by MATLAB will be saved in.

Arithmetic, Variables and Errors

At its most basic level, MATLAB is just like a calculator. Things should behave as you might expect. Type in a mathematical expression, press enter and the answer is calculated and displayed:

```
>> 1+2
ans =
      3
```

```
>> 2*6
ans =
      12
```

```
>> 2/4
ans =
      0.5
```

```
>> 7^2
ans =
      49
```

Notice that we use * for multiplication, / for divide and $\wedge$ for raising to the power.

A **variable** is like a box in computer's memory, used to store something. For example, you can create a variable called `x` and store the number 7 in it:

```
>> x = 7
```

This is called an **assignment**. It assigns the value 7 to the variable x. It allocates space in memory to store a number, then it puts 7 into that space. In general an assignment consists of a variable name on the left, followed by an equal sign with an expression on the right: $variable = expression$. Typing 7 = x for example, will give an error as you cannot assign the value 'x' to the number 7. All the variables you have created will be stored in the **workspace**. Typing a variable name will display the value stored in it to screen. Typing `whos` will show all the variables you have stored.

The variable `x` can then be used in other **expressions**. The value of the expression is calculated and the result is stored into the variable. For example, by entering

```
>> y = x^2 + 5*x -3
```

variable y will be assigned a value of 81 in the workspace.

You can assign a new value to an existing variable, by repeating the assignment with a different number. The last command you type will always override an earlier assignment. For example typing $y = 5$ will override the y expression you assigned above.

```
>> y = 5
```

Notice 81 is no longer in the workspace as it has been overwritten.

MATLAB is case sensitive. This means x and X are different variables. Variable names are a single word and can contain underscores.

Computer programs have to be very clear and precise for the computer to understand what it is you want. They have their own computer language that is much stricter than written English. When you get an error message, it is very important that you read the error message and try to understand what it is telling you. For example, an error as MATLAB needs something next to brackets:

```
>> 3(1+2)
Error: Unbalanced or unexpected parenthesis or bracket.
```

The following will not produce an error:

```
>> 3*(1+2)
```

Error messages are common when programming - do not worry! Try to understand the message, it will help you solve the problem!

Built-in Functions

Just like on a scientific calculator, there are **functions** that you can use in your calculations. Functions are **called** and **act on** a value or variable that has been **passed** into it. As an example, we can use the square root function by typing the following into the command window:

```
>> x = 2
>> y = sqrt(32*x)
```

The steps the computer takes to **evaluate a function call** are:

- The expression in the brackets is evaluated.
- The value of the expression is passed to the function input.
- The function returns the square root of the input.
- The output of the function is written into the variable y.

The `sqrt` function does not know anything about the variable `x`, it only sees the value `64` as the input and it returns the value `8`.
Some other examples of functions:

```
>> d = 4*sin(pi/2)
>> l = log(50*d)      % Finds natural logarithm of 50*d
>> r = randi(100)     % Generates a random integer between 1 and 100
```

Functions can also take **multiple inputs** e.g. `rem(a,b)` finds the remainder of a/b. So `rem(6,3)` outputs `0` and `rem(9,4)` outputs `1`.

A key feature of MATLAB is the ability to write your own functions. You will be taught how to do this in your first year.

Help Documentation There are so many things that MATLAB is capable of doing that the software can seem overwhelming upon first use. Luckily there is lots of documentation to help you along the way. You can type `help` followed by a function name to find out more about it and search for terms in the documentation using `doc`:

```
help isprime
doc prime
```

Data Types Variables come in different **data types**. Above we looked at some numbers, which are stored as *doubles* - a way of representing numbers. As well as numbers, we can declare a variable as a **string** of text:

```
myName = 'Sally'
```

Booleans and Flow Control

Another data type is a **Boolean** also known as a **logical**. A variable that is a Boolean only has two possible values: 1, representing 'true' or 0, representing 'false'. For example the function `isprime(x)` returns logical `1` if `x` is prime, and logical `0` if `x` is not prime. Below are the results from the prime checking function on two values:

```
>> p1 = isprime(7)
p1 =
   logical
     1
```

```
>> p2 = isprime(10)
p2 =
   logical
     0
```

Logicals are also created when an expression with a **relational operator** is declared. Example logicals:

```
>> 10>100
ans = logical
          0
```

```
>> 4>=3
ans = logical
          1
```

```
>> 4~=3
ans = logical
          1
```

```
>> 1==2
ans = logical
          0
```

Available operators are shown in the Table below:

Relational Operator	Notation
Less than	$<$
Greater than	$>$
Less than or equal to	$<=$
Greater than or equal to	$>=$
Not equal to	˜ $=$
Equal to	$==$

Expressions can be assigned to a variable:

```
>> check1 = (4 < 3)
check1 =
  logical
   0
```

The expression is false (4 is not less than 3), so a variable `check1` is stored is the workspace with logical value `0`.

Flow Control gives us the ability to choose different outcomes based on what else is happening. Flowcharts are an intuitive way to understand flow control in an algorithm. Figure 2 shows how different outcomes occur in a program based on *if* x is prime.

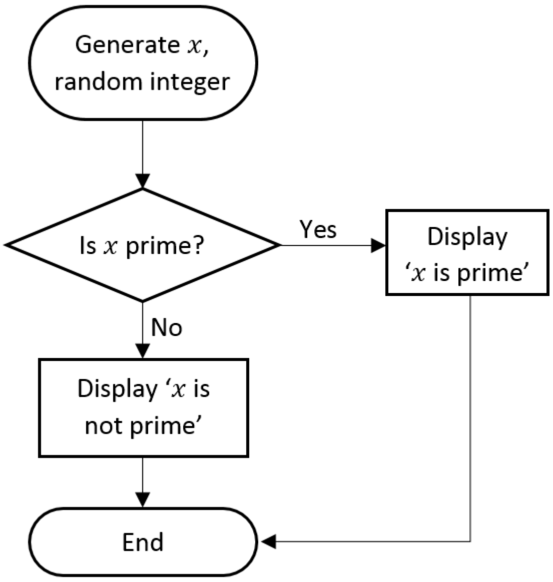


Figure 2: Check if prime

We use **IF-ELSE-END** statements to code flow control:

General Form:

IF statement == true

do something

ELSE

do something else

END

Code for the algorithm in Figure 1:

```
x = randi(50)
if isprime(x)==1
    disp('x is prime')
else
    disp('x is not prime')
end
```

Demos

It is highly likely that this is your first time using MATLAB. We have started with the very basics. Do not let this make you underestimate what MATLAB is capable of. The demos below explore some applications that are possible using MATLAB.

Vibrating Membranes: Typing `vibes` into the Command Window calls a demo which solves the wave equation for the vibrations of an L-shaped membrane, and then plots the results. The L-shaped geometry is of particular interest mathematically because the stresses approach infinity near the reentrant corner. You will learn how to create solvers like this in your second and third year.

Bending Truss: The `truss` demo animates 12 natural bending modes of a two-dimensional truss.

Chaotic Systems: The `lorenz` demo animates the integration of three differential equations that define the ”Lorenz Attractor”, a chaotic system. As the integration proceeds you will see a point moving around in a curious orbit in 3-D space known as a strange attractor. The orbit is bounded, but not periodic and not convergent (hence the word ”strange”).

For fun try some of the following commands:

```
why
fifteen
xpbombs
```

Quiz

1. We assign the value 9.81 to `g`, and 50 to `m`:

```
g = 9.81
m = 50
```

What is the correct way to calculate `m` multiplied by `g` and assign it to a variable named force using MATLAB syntax?

   A. `force = mg`

   B. `mg = force`

   C. `force = m x g`

   D. `force = m*g`

2. What values are assigned to `x` and `y` after these statements have been executed in this order in the command line?

```
>> x = 3
>> y = 5
>> x = y
>> y = x
```

3. Booleans are logical expressions which are true (`1`) or false (`0`). Identify the values of the logical expressions below:

Expression	Value of logical
`4 == 3`	
`5 ~= 3`	
`4 >= 1`	
`3.2 <= 10.1`	

4. Boolean algebra is the combination of logical expressions. This is useful for when we need to check multiple conditions, which we often need to do when programming.

The chart on the right shows how multiple logcial expressions are combined in a computer to create one logical. The **AND** operator in MATLAB is denoted `&&`, whereas **OR** is denoted `||`.

```
>> x = 5;
>> (x<10) && (x>6)
ans = logical
        0
>> (x<10) || (x>6)
ans = logical
        1
```

$(x < 10)$ and $(x > 6)$

$\downarrow$

True and $(x > 6)$

$\downarrow$

True and False

$\downarrow$

False

$\downarrow$

logical 0

Fill in the Boolean logic table below. You can always check your answers by setting up two statements in MATLAB.

Expression	Evaluates to...
True AND True	
False AND False	
True AND False	
False AND True	
True OR True	
False OR False	
True OR False	
False OR True	

5. What MATLAB code will give a logical 1 when a variable `x` is even?
(Hint: Look back at the `rem` function and the == operator).

6. An algorithm is a finite sequence of precise instructions to solve a problem. They can be communicated in various ways, for example written English, pseudo code or flowcharts. Consider the flow chart shown in Figure 3. Do you notice anything unusual about it? Test some different numbers and see what messages are displayed.

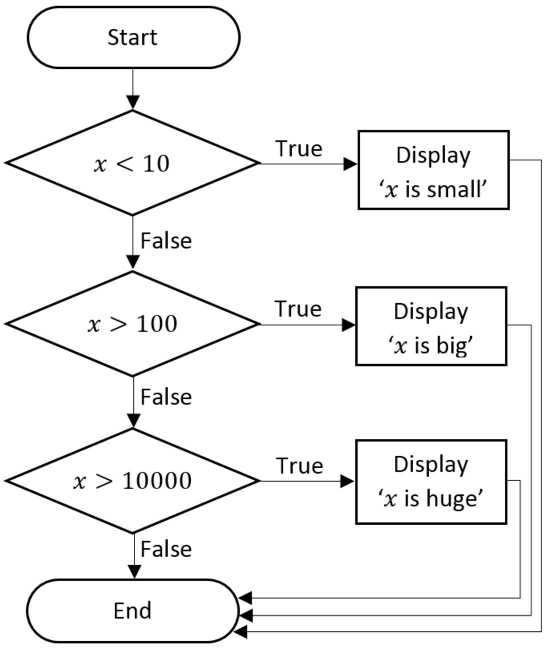


Figure 3: Flowchart for number sizing

7. Write an algorithm to check if a random integer `x` is odd or even. You can write out a method as a series of steps, draw a flow-chart or use pseudo-code.

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UNIVERSITY OF OXFORD

DEPARTMENT OF ENGINEERING SCIENCE

DEPARTMENT OF ENGINEERING SCIENCE

UNDERGRADUATE INDUCTION DAY
Friday 6th October 2017

Undergraduate induction will take place in the Engineering Science Department Thom Building on Friday 6th October 2017. You should aim to arrive at Lecture Room 1 on the 1st floor by 1.55pm. **The induction programme will start promptly at 2.00pm**.

The afternoon will consist of two parts:

Part I	**Welcome and Introductions – LR1**
2.00pm	Welcome to the Department of Engineering Science *Professor Lionel Tarassenko, Head of Department*
2.20pm	Introduction to Student Support *Dr Joanna Rhodes, Head of Finance and Administration*
2.30pm	Introduction to Safety in the Department of Engineering Science *Mr Gary Douglas, Safety Officer*
2.40pm	Introduction to IT Support *Mr Gareth Edwards, IT Manager*
2.50pm	Welcome from the Faculty Office *Dr Stephen Payne, Associate Head (Teaching)*

Part II

Registration with the Department

3.10 – 4pm Collect course materials from the vestibule area outside LR1 and LR2.

To: New 1st Year Engineering Students

From: Chair of the Faculty of Engineering Science

Use of Calculators in Engineering Examinations

As specified in the University's **Examination Regulations**, in your Preliminary examinations you will be permitted to take into the examination room *one* calculator of the types listed below:

- **CASIO fx-83 series (e.g. Casio FX83GT)**
- **CASIO fx-85 series (e.g. Casio FX85GT)**
- **SHARP EL-531 series (e.g. Sharp EL-531WB)**

You are advised to buy a calculator of the type listed above in good time, and to familiarize yourself with its operation before your Preliminary examinations.

Please note that the restriction will apply to **examinations** only. For all of your laboratory, project and tutorial work, you are free to use any calculator you wish.

DEPARTMENT OF ENGINEERING SCIENCE

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UNIVERSITY OF OXFORD

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I agree to be photographed/filmed. I also agree that such material may be kept in the Department's media resources library database and in the University Photographic Library, for use in Departmental/University publications, websites, lectures and events and may also be passed on to third parties for use in *bona fide* publications. I am over the age of 18.

Signed …………………………………………………………… **Date** ……………………

Name (in capitals) ……………………………………………… **College** ………………….

If the student is under the age of 18 years, the following section must be completed.

I herby certify that I am the parent/guardian of …………………………………………………..
and on their behalf give my consent without reservation to the aforementioned.

Signature of parent or guardian ………………………………………. **Date** ………………

Name (in capitals) ……………………………………………

Address

Under the Data Protection Act 1998, your photograph constitutes personal data, and as such will be kept in accordance with the provisions of that Act. If you wish to object to the use of your data for any of the above purposes, please give details here.

FIRST YEARS: Please bring the completed form to the Department of Engineering Science Induction.

OXFORD UNIVERSITY DEPARTMENT OF ENGINEERING SCIENCE

Application for computer resources on departmental facilities

Name ……………………………………………………………………………….

Course ……………………………………………………………………………….

College ……………………………………………………………………………

College Tutor …………………………………………………………………..

I accept that all software systems and software packages used by me are to be regarded as covered by software licence agreements, with which I agree to abide. Unless specifically stating otherwise this agreement will prohibit me from making copies of the software or transferring copies of the software to anyone else, other than for security purposes, or from using the software or any of its components as the basis of a commercial product or in any other way for commercial gain. I indemnify the Chancellor, Masters and Scholars of the University of Oxford, and the Oxford University Department of Engineering Science, for any liability resulting from my breach of any such software licence agreement.

I will not use personal data as defined by the Data Protection Act on computing facilities made available to me in respect of this application other than in the course of my work as per the University's registration. I accept that the Oxford University Department of Engineering Science reserves the right to examine material on, or connected to, any of their facilities when it becomes necessary for the proper conduct of those facilities or to meet legal requirements and to dispose of any material associated with this application for access to its resources upon termination or expiry of that authorisation.

I agree to abide by any code of conduct relating to the systems I use and the University policy on data protection and computer misuse -http://www.admin.ox.ac.uk/statutes/regulations/196-052.shtml

In particular, I will not (by any wilful or deliberate act) jeopardise or corrupt, or attempt to jeopardise or corrupt, the integrity of the computing equipment, its system programs or other stored information, nor act in any way which leads to, or could be expected to lead to, disruption of the approved work of other authorised users.

Signature ……………………………………………….. Date ……………………….

FIRST YEARS: Please bring the completed form to your first computing practical.